Report from the CA Framework Committee

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The Big News:
The Framework gives teachers, mathematics departments, and district mathematics specialists local control over:

- what textbooks to buy
- how to teach
- how to organize high school mathematics content
- how and when to accelerate
Revising the CA Additions

- Removal of Algebra 1 as default 8\textsuperscript{th} grade math class.

CCSS 8\textsuperscript{th} grade is now the default class at 8\textsuperscript{th} grade. Algebra 1/Integrated 1 is now a 9\textsuperscript{th} grade class and is DIFFERENT than what Algebra 1 was under the old standards.
CA Changes to CCSS

- Reworded and relocated CA additions to reflect CCSS Language and Intentions
- G-GMD-6. Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length; apply these relationships to solve real-world and mathematical problems.

(Formerly G-CO.10.1: Know and use the triangle inequality theorem.)
CA Changes to CCSS

* Redundancy of CA additions
* 2-MD-3.1 Verify reasonableness of the estimate when working with measurements (e.g., closest inch). (CA-Standard NS 6.1)
  * Removed because this standard is part of the SMPs.
High School “coursification” in the Framework is an example and not a mandate.

“The pathways and courses are models, not mandates. They illustrate possible approaches to organizing the content of the CCSS...States and districts are not expected to adopt these courses as is; rather, they are encouraged to use these pathways and courses as a starting point for developing their own.”

CCSS, Appendix A, p.2
High School Coursification

“Integrated”
Mathematics 1
Mathematics 2
Mathematics 3

“Traditional”
Algebra 1
Geometry
Algebra 2

“Customized”
Career/Tech Courses
(eg. Mathematical Modeling)

Precalculus

Statistics & Probability

AP Calculus

AP Statistics
b. Solve quadratic equations by inspection (e.g., for \( x^2 = 49 \)), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \( a \pm bi \) for real numbers \( a \) and \( b \).
Parenthetical notations in cluster headings will help clarify what part of standard is required or the depth necessary at given level.

“...quadratics with real solutions.”
Pros & Pros

**Integrated**

- Addresses the typical repetition of Algebra 1 at start of Algebra 2.
- Matches the CCSS integrated approach for K-8 mathematics and the approach in most other countries.
- Real-world problems often blend algebra, functions, geometry, probability and statistics.
- Students taking SAT will have had more geo content more recently.

**Traditional**

- Parents, staff, administrators are familiar with the separation of algebra and geometry.
- Possibly lends itself better to “doubling up” for acceleration.
- “Math 1” faculty may not be mathematically qualified to teach the geometry mixed into the algebra.
High School Mathematics Integrated Pathway

What are arguments worth considering in support of selecting an integrated approach to high school mathematics?

The typical barriers to adopting an integrated pathway are:
- Lack of familiarity
- Lack of quality resources (textbooks, units)
- Poor examples of implementation
“The authors examined curricular effectiveness in high schools that offered parallel paths...an integrated approach or a (traditional) subject-specific approach...Students in **the integrated curriculum scored significantly higher** than those in the subject-specific curriculum on the standardized achievement test.”

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The Effects of Content Organization and Curriculum Implementation on Students’ Mathematics Learning in 2nd-Year High School Courses

Tarr, Grouws, Chávez, and Soria, NCTM Journal for Research in Mathematics Education, July 2013
### San Diego County
- Oceanside USD
- San Diego USD
- San Dieguito Union HSD
- Vista USD

### Stanislaus County
- Ceres
- Modesto City
- Oakdale
- Turlock

### San Bernardino County
- Chaffey Joint Union HSD
- Redlands
- San Bernadino

### San Joaquin County
- Escalon USD
- Ripon USD

### Santa Barbara County
- Santa Ynez Valley

### Santa Clara County
- Campbell Union HSD
- East Side Union HSD

### Tulare County
- Alpaugh USD
- Burton SD
- Dinuba USD
- Exeter USD
- Lindsay USD
- Porterville USD
- TCOE – La Sierra

### Shasta County
- Shasta Union HSD
- Gateway USD

### Tuolumne County
- Big Oak Flat Groveland USD
- Sonora Union HSD
- Summerville Union HSD

### INTEGRATED STATES
- North Carolina
- Utah
- West Virginia

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Incomplete list of Integrated Districts
Acceleration

History

Data

International Benchmark
Acceleration

“When students have mastered the content described in the Common Core State Standards for Mathematics with California Additions for kindergarten through grade eight, they will be ready to complete Common Core Algebra I or Common Core Mathematics I.”

-CDE Math Framework Draft
Compacting in Middle School

Grade 6

Grade 7 + Part of Grade 8

Part of Grade 8 + Algebra I or Integrated I

Geometry or Integrated II

Algebra II or Integrated III

Precalc

Calculus (AP)

ADP

Doubling Up in High School (only apply to traditional pathway)

Grade 6

Grade 7

Grade 8

Algebra I

Algebra II

Precalc

Calculus (AP)

Geometry

ADP
Accelerated Integrated Pathway – 3 years into 2

Grade 6 → Grade 7 → Grade 8 → Integrated I
And Part of Integrated II → Part of Integrated II and Integrated III → Precalc → Calculus (AP)

Enhanced Pathway – 4 years into 3

Grade 6 → Grade 7 → Grade 8 → Enhanced Algebra I/Integrated I → Enhanced Geometry/Integrated II → Enhanced Algebra II/Integrated III → Calculus (AP)

ADP

Content of pre-calc delivered in the enhanced courses
To Accelerate, or Not, Huff Post Education, Hung-Hsi Wu, Professor of Mathematics Emeritus at the University of California at Berkeley (Posted: 09/20/2012 3:42 pm)

E-Mails between Dr. Jacob and Sunny in August, 2013

California CCSSM Mathematics Framework (Draft) Appendix E: Higher Mathematics Pathways Standards Chart

MAA and NCTM position statement
As far as taking AP tests, it is more important that students take an all around challenging curriculum than race to and extra math AP course. In admission to UC the readers are aware of AP courses, but not so much the test scores, and so if the student had honors courses that lead to an AP course in grade 12 that is going to be about as strong as they can do. If this rush leads to weaker grades overall the student is much worse off. What I’ve seen over the past 25 years teaching at UCSB is that the BC Calculus really doesn’t move students much further forward mathematically (perhaps one quarter, ten weeks, at most) and I would much rather see them have a thorough preparation including applications of geometry in the years prior to calculus. The students who don’t have these preliminary experiences have tremendous difficulty in my differential equations and multivariable calculus courses (certainly those in my classes when I interview them). I know it is hard to persuade parents that their students shouldn’t race to get calculus, but I really wish they wouldn’t. So I like your plan. Keep up the good work.
...the ultimate goal of the K–12 mathematics curriculum should not be to get students into and through a course in calculus by twelfth grade but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college.

Recommendations for Districts
Implications for Elementary Teachers

- Mathematical content standards in the elementary grades have some changes from current standards which reflect the need for student understanding.

- Mathematical practice standards address the “habits of mind” that students should develop to foster mathematical understanding, expertise, skills, and knowledge.
Implications for Elementary Teachers

- A sea change in thought about teaching math: “a mile deep and an inch wide, instead of an inch deep and a mile wide” in our current standards.
- Not teaching content standards in isolation.
- Emphasis on problem-solving, using mathematical reasoning and not “strategies”, like looking for word clues.
Implications for Elementary Teachers

- Need for stronger content-area knowledge.
- Need for a bigger “tool box” of pedagogical skill.
- Continuing articulation between grade levels to provide teachers with clear understanding of each step of children’s mathematical growth.
Implications for Elementary

- Student work should be longer-term problems which integrate several content standards and MP standards.
Implications for Elementary
Implications for Elementary

- Conceptual understanding is the key
- Guiding students with a more developmental approach (i.e., Bruner’s modes of representation) to deep understanding:
  - Manipulative
  - Representative
  - Symbolic
Implications for Elementary

- Manipulative
- Representative
- Symbolic

These modes are addressed by integrating content, developing several concepts at the same time at different points of understanding, not all in the same lesson.
Integrating Content in a Developmental Model

Preparing a lesson set which integrates contents and mathematical practice standards with a developmental approach:

- **Manipulative**: Using base ten blocks to build two-digit numbers and subtract.
- **Representational**: Playing Chip Trading (subtraction with regrouping)
- **Symbolic**: Addition with regrouping used to solve pattern puzzles in a hundreds chart
Integrating Content in a Developmental Model

- Current 3 step methods concentrate on one concept/procedure, doing all three parts in one lesson.

- This model is an overlap of concept/procedure over a period of time, developing deep understanding of a new concept while working symbolically with a previous concept.
TENS AND ONES TOSS UP
**TENS AND ONES TOSS UP**

- **Materials:** 9 prepared wooden craft sticks, paper/pencil. Optional: 1 penny for each pair.
- **First,** make sets of nine sticks, with ten dots on one side, and one dot on the other. Band or bag them. Give a set to each pair. In pairs, each player gently tosses the sticks and names the number tossed ("Four tens and five ones is forty-five"). The player with the greater number gets a point. The player at the end of time with the most points wins.
VARIATION: Players toss and name their number, and then toss penny. Heads means greater number wins, tails, least number wins.
Implications for Secondary: WALK AWAY

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>Distance between Walker and CBR (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Implications for Secondary:
WALK AWAY

1. In terms of the walker, what is the meaning of the first point?

2. On average, how fast was the walker walking?
Implications for Secondary: WALK AWAY

3. Complete the chart below as if the walker started at the same place and walked constantly at the speed you calculated in #2.

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>Distance between Walker and CBR (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Implications for Secondary: WALK AWAY

4. Suppose the walker continued to walk at the constant speed you calculated in #2. How far from the CBR would they be after 10 seconds?

5. Describe an equation you could use to compute how far the walker would be from the CBR after walking for $t$ seconds. Write the equation.
RIGOR

- Applications
  - Can often be used to build
- Conceptual Understanding
- Fluency

Before
Multiple Representations

- Numeric
- Pictorial
- Symbolic
- Verbal

NCTM Representation Principle & MP 2
Multiple Representations

* The National Math Panel Report indicates that students benefit from experiencing **contextualized representations** before **abstract representations** (CRA Instruction).

* According to the National Math Panel Report, research indicates that students should be **dealing with multiple representations** (symbolic, verbal, pictorial, numeric) **within the same unit of instruction**.
Math Practice 4: Not Just Word Problems

Real Word Problems
Publisher’s Criteria

- Publisher’s Criteria is used to guide publishers how to write a book that will make it onto CAs list of approved books.

- It guided reviewers on what materials made the approved list in CA.

- This adoption is a SOFT adoption: districts can purchase a book on or off the list! First time this has ever happened!!!
Publisher’s Criteria: Category 1

2. The materials in basic instructional programs support comprehensive teaching of the Common Core State Standards for Mathematics with California Additions and include the standards for mathematical practice at each grade level or course.

6.a. Developing students’ conceptual understanding of key mathematical concepts, where called for in specific content standards or cluster headings, including connecting conceptual understanding to procedural skills.
Focus, Coherence, and Rigor

8.b. Including problems and activities that serve to connect two or more clusters in a domain, or two or more domains in a grade, in cases where these connections are natural and important.

10. Focus and Coherence via Practice Standards: Materials promote focus and coherence by connecting practice standards with content that is emphasized in the Standards.

12. Emphasis on Mathematical Reasoning: Materials support the Standards’ emphasis on mathematical reasoning
Sample Pages from Texts

- Some texts were written prior to CCSS
- Some texts claim to integrate the SMPs
- Some have always incorporated the SMPs
- Some texts are new from the ground up
- Use the Publisher’s Criteria to sort out differences in the texts
11.3 Subtracting Integers

**Essential Question** How are adding integers and subtracting integers related?

1. **ACTIVITY:** Subtracting Integers

   Work with a partner. Use integer counters to find $4 - 2$.

   - Start with 4 positive counters.
   - Remove 2 positive counters.
   - What is the total number of counters?

   $4 \quad 4 - 2 \quad -$  
   
   So, $4 - 2 = -$  

2. **ACTIVITY:** Adding Integers

   Work with a partner. Use integer counters to find $4 + (-2)$.

   - Combine 4 positive counters and 2 negative counters.
   - Remove zero pairs.
   - What is the total number of counters?

   $4 \quad + \quad -2 \quad = \quad 4 + (-2) \quad -$  

   So, $4 + (-2) = -$  

3. **ACTIVITY:** Subtracting Integers

   Work with a partner. Use a number line to find $-3 - 1$.

   - Then move 1 unit left to end at  
   - Subtract 1. 
   - Start at 0. Move 3 units to the left.

   So, $-3 - 1 = -$
Check Your Understanding

For Exercises 1–3, copy each number line on graph paper. Label the missing points.

1. 
   0 ■ 2 5 8
   ■ 7 7 7 7

2. ■
   1 2
   ■ 1 2

3. ■
   ■ 1
   ■ 2
   ■

4. How many thirds are in each number?
   a. \( \frac{5}{3} \)
   b. \( \frac{25}{3} \)
   c. 3
   d. \( \frac{14}{3} \)
   e. 17
   f. \( \frac{12}{6} \)
   g. \( \frac{5}{6} \)
   h. 69
   i. 11

5. What's Wrong Here? Rosa draws this number line incorrectly. Explain why this is not an accurate number line. Correct the number line.

6. Place the following numbers on a number line.
   \( \frac{3}{2}, 4, \frac{5}{2}, \frac{7}{4}, \frac{5}{6}, \frac{7}{6}, \frac{9}{6}, \frac{10}{6}, \frac{11}{6} \)
   a. Which fractions represent the same number, or are equivalent?
   b. What is the least number in this list?

7. Draw a picture that illustrates the equivalence \( \frac{3}{5} = \frac{4}{10} \).

On Your Own

8. Find four different fractions equivalent to \( \frac{7}{3} \).

9. A \( \frac{3}{8} \)-inch wrench is slightly too small to fit a bolt on your lawn mower. You have three more wrenches that measure \( \frac{7}{16}, \frac{11}{16}, \) and \( \frac{3}{16} \) inches. Which wrench should you try next? Explain.
High School – Parallel Lines

3-2 Properties of Parallel Lines

Objectives
To prove theorems about parallel lines
To use properties of parallel lines to find angle measures

Getting Ready!

In the Solve It, you identified several pairs of angles that appear congruent. You already know the relationship between vertical angles. In this lesson, you will explore the relationships between the angles you learned about in Lesson 3-1 when they are formed by parallel lines and a transversal.

Essential Understanding
The special angle pairs formed by parallel lines and a transversal are congruent, supplementary, or both.

Postulate 3-1 Same-Side Interior Angles Postulate

If \( \ell \parallel m \) then same-side interior angles are supplementary.

\[
\begin{align*}
\text{If } & \ell \parallel m \\
\text{Then } & m \angle 4 + m \angle 5 = 180 \\
& m \angle 3 + m \angle 6 = 180
\end{align*}
\]
Geometry – Symmetry

Tilings with Triangles or Quadrilaterals

The figures below show portions of tilings or tessellations of equilateral triangles and squares. The tilings are made of repeated copies of a shape placed edge-to-edge so that they completely cover a region without overlaps or gaps.

1. Assume that the tilings are extended indefinitely in all directions to cover the plane.
   i. Describe the various ways that you can slide a tracing of each tiling so that it coincides with the original tiling. These tilings have translation symmetry.
   ii. How could you describe the translation symmetries using arrows?
   iii. Do the extended tilings have any reflection symmetry? If so, describe the lines of symmetry.
   iv. Do the extended tilings have any rotational symmetries? If so, describe the centers and angles of rotation.

b. For these two tilings:
   i. what is the sum of the measures of the angles at a common vertex?
   ii. what is the measure of each angle at a common vertex?

c. In the tiling with equilateral triangles, identify other common polygons formed by two or more adjoined triangles that also produce a tiling. Sketch each and show the equilateral triangles that form the new tile. What does this suggest about other polygons that could be used to tile? Explain your reasoning.

2. Now explore if other triangles can be used as tiles.

   a. Working in groups, each member should cut from poster board a small triangle that is not equilateral. Each member's triangle should have a different shape. Individually, explore whether a tiling of a plane can be made by using repeated tracings of your triangle. Draw and compare sketches of the tilings you made.

   b. Can more than one tiling pattern be made by using copies of one triangle? If so, illustrate with sketches.

   c. Do you think any triangle could be used to tile a plane? Explain your reasoning. You may find software like the "Tilings with Triangles or Quadrilaterals" custom app helpful in exploring this question.
Geometry – Transformations

Getting Started

There are different types of transformations in a plane. A reflection suggests a mirror. But you can also experiment with reflections by folding paper.

In-Class Experiment

On a piece of plain paper, draw a capital letter $F$ an inch or two high. Here is how to find the image of your letter $F$ (the preimage) after reflection over a fold line.

- Fold the paper to cover the letter. The crease, or fold line, can be anywhere you like, but it should not intersect the letter.
- Trace the letter onto the back of the paper.
- Unfold the paper. Trace your tracing onto the front of the paper. Also, draw along the crease to show the fold line better.

This new picture is the image of your original picture after reflection over your fold line. Place a mirror along the fold line and look into the mirror from the side with the preimage. You will see this image in the mirror.

- Label the image with an uppercase letter different from $F$. Label the fold line with the corresponding lowercase letter.
- Choose some other lines of reflection and repeat this process using your original preimage $F$. You can have one or two fold lines intersect $F$.
- Visualize the process to avoid lines that will reflect the preimage off your paper and predict where the image will be on the paper. Check each prediction.

For You to Explore

1. On a plain piece of paper, use a ruler to draw a line segment $\overline{AB}$. Fold the paper by matching point $A$ to point $B$. Call the fold line $\ell$. Call the point where $\ell$ and $\overline{AB}$ intersect point $X$.
   a. Which segment has greater length, $\overline{AX}$ or $\overline{BX}$? Explain.
   b. Choose a point $C$ on $\ell$ but not on $\overline{AB}$. What is the measure of $\angle BXC$?
   c. Describe $\triangle ABC$. Does it have any special characteristics?
   d. Describe the relationship between $\ell$ and $\overline{AB}$. 
Dividing Polynomials

1. Divide polynomials using long division.
2. Divide polynomials using synthetic division.

**Example 1** Divide a Polynomial by a Monomial

\[ \frac{6x^3y^2 + 12x^2y^2 - 18x^2y}{3xy} \]

Simplify:

- \(6x^3y^2 \div 3xy = 2x^2y^2\)
- \(12x^2y^2 \div 3xy = 4xy\)
- \(-18x^2y \div 3xy = -6x\)

**Guided Practice**

1A. \(20x^3y^2 - 16cdy^2 + 4cdy^2 \div (4/3)\)
1B. \((18x^2y + 27x^2y^2 + 3xy)^{-1} \cdot 6x + 9x^2yz\)

You can use a process similar to long division to divide a polynomial by a polynomial with more than one term. The process is known as the division algorithm.

**Example 2** Division Algorithm

Use long division to find \((x^2 + 3x - 40) \div (x - 5)\).

\[
\begin{array}{c|cc}
& x + 8 & \\
\hline
x^2 & 5x^2 & + 3x & - 40 \\
\hline
-5x & -5x^2 & - 25x & + 200 \\
\hline
0 & 0 & + 60 & - 60 \\
\hline
0 & 0 & + 60 & - 60 \\
\hline
\end{array}
\]

The quotient is \(x + 8\). The remainder is 0.

**Guided Practice**

2A. \((x^2 + 2x - 30) \div (x - 3)\)
2B. \((x^2 - 13x + 12) \div (x - 1)\)
A-G Compliance
*
Either Framework coursification will be accepted by UC as A-G compliant.

Intersegmental News
*
Common Core deemed to be compliant with Statement on Competencies for incoming freshmen
UC/CSU/CCC News

* EAP
  * SBAC will be using the 11th grade assessment to create new EAP which CSU will use to place incoming freshman

* BOARS
  * Practice Standards at least as important as Content Standards
CCTC News

- CSET
  - Multiple Subject and Single Subject standards rewritten relative to Common Core

- Subject Matter Waiver Programs need to complete matrix indicating where they are covering mathematics needed to train new teachers relative to Common Core