Ideas Worth Sharing: Conceptually-Based Learning

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Lorie Sousa, Ph.D., Key Data Systems

Developing Educators Learning to Teach Algebraically
What do you know about the sum of the measures of the interior angles of polygons?
1. Draw a straight line on a piece of paper.
2. Use a pencil to shade the vertices of your triangle.
What is triangulation?

Drawing diagonals from any vertex to form triangles that make up the interior angles of the polygon
Who We Are

Project DELTA, a Cohort 7 CaMSP

Partners:
Riverside County Office of Education (RCOE),
CSU San Bernardino (CSUSB),
Perris Union High SD
Perris Elementary SD
Romoland Union SD
Nuview Union SD
Menifee USD

Participants: 73 teachers of mathematics in grades 3-Alg 1
To share reflections on our participants' and professional development (PD) providers' experience using coursework from the Math Content Program for Teachers (MCPT).

To share the data and results from this program.
Outline

What is MCPT?

Teachers' Positive Takeaways

Teacher and Student Data

Challenges

Next Steps
Teachers are the key to changing the way students learn mathematics.

~Dana & Yendol-Silva, 2003
• Teachers need content instruction on a weekly basis
  • Links content to the classroom
  • Flowed into the content taught to all DELTA teachers in PD

• Links between content and pedagogy

• Best practice by UCLA

• Recognized by other CaMSP projects
  • Math content courses
  • Written into the Project DELTA grant
In order to increase the number of highly-qualified teachers in partner LEAs, we offered 32 quarter-units of coursework applicable to a Supplemental Authorization in Mathematics. The Curriculum for the courses is the Mathematics Content Program for Teachers, developed and used successfully at UCLA and approved as a program leading to a Supplemental Authorization to teach Mathematics.
Project DELTA Design Diagram

Spring
Kick Off Event
May 26, 2010

July
Intensive 8-days/2 weeks (M-Th, M-Th)

Sept – Nov.
Site/Grade level based lesson study days
(specific dates TBD by specific lesson study groups)

Jan. – Mar.
Intensive Saturday Sessions
Jan. 22, 2011
Feb. 5, 2011

Feb. – May
Site/Grade level based lesson study days
(specific dates TBD by specific lesson study groups)

June
Culminating Event

Numbers & Expressions

July 19-22
July 26-29

60 hrs

July 2011 – June 2012

Proportional Reasoning

July 18-21
July 25-28

60 hrs

July 2012 – June 2013

Functions & Equations

July 16-19
July 23-26

8 quarter-units MCPT coursework*

* For participants seeking Mathematics Supplemental Authorization or Subject Matter Authorization
A course sequence developed at UCLA to develop strong conceptual foundations for the mathematics taught in grades K-12.
MCPT Course Sequence

- Math Reasoning C
- Number Power 1
- Number Power 2
- Perspectives on Algebra
- Perspectives on Geometry
- Topics in Algebra and Geometry
- Dealing with Data
- Perspectives on Functions 1
- More Topics in Geometry
- Math and Technology
- Perspectives on Functions 2
- More Topics in Data
- More Topics in Algebra
Teachers gain experience learning through solving meaty problems and sharing multiple solution strategies and representations.

Pedagogical considerations and strategies come to the surface through discussion of content.
Class Design

- Weekly Classes
- Surveyed teachers for best day
- Central location to school sites
- Cohort design
Grant PI and Co-PI taught some of the courses to ensure flow of content to PD for all grant participants.

UCLA staff trained our instructors to ensure fidelity to the curriculum.

Image credit: Helen Chan, http://www.math.ucla.edu/mcpt/
If I had the opportunity to take MCPT courses again, I would do it in a heartbeat. If I had the power to make one change, I would urge my coworkers to participate as well. Learning alongside colleagues has been the most gratifying experience of my career.

Developing as a member in a team of learners makes me feel that I am a part of something special, a part of something unbreakable.

The fact that we worked with the same group of people for 80 weeks helped us form supportive and collegial alliances that will last throughout our careers and beyond.
Why Weekly Meetings?

The way we discussed and dissected math concepts provided excellent practice and reflection for doing the same with students.

The way we approached the activities using multimodal strategies helped reinforce the concepts and provided examples for us to bring back to our classrooms.
The “fourfold way” is a collection of four ways of approaching a math problem, solving it, and expressing the solution.

- **Numbers** – Express solution numerically
- **Pictures** – Express solution visually
- **Symbols** – Express solution algebraically
- **Words** – Express solution verbally or in writing

Taken together these four ways should lead to a complete and comprehensive solution.
Fourfold Way

**NUMBERS**
Express the solution numerically.

**PICTURES**
Express the solution visually.

**SYMBOLS**
Express the solution algebraically.

**WORDS**
Express the solution verbally or in writing.
Mary wants to buy a new DVD player and the one she wants costs $150.00. Mary makes $20/hour for babysitting her 4 mean nephews. How many hours will she have to baby sit in order to pay for the DVD player?

\[ 20x = 150 \]

\[ \frac{20x}{20} = \frac{150}{20} \]

Mary wants to buy a new DVD player and the one she wants costs $150.00. Mary makes $20/hour for babysitting her 4 mean nephews. How many hours will she have to baby sit in order to pay for the DVD player?

How many apples, each weighing 2 ounces, will be needed to balance three 2-pound weights?
My thinking has changed as I have learned to see every problem in the four-fold way. I try to look for a pictorial representation where I would never even attempt that before.
Links Between Content and Pedagogy

Algebra Walk Classroom Connections

Graphing Standards

Previous Grade Level: (used 6th grade standards)
- NS 2.3 Identify one more than, one less than, 10 more than, and 10 less than a given number
- MR 1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
- MR 1.2 Determine how and when to break a problem into simpler parts
- MR 2.5 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work
- MR 3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
- AF 1.1 Use information taken from a graph or equation to answer problems about a problem situation.
- AF 1.2 Write and evaluate an algebraic expression for a given situation for up to 3 variables

My Grade Level: (used grade 7/8)
- NS 1.2 Use words, models, and expanded forms (e.g., 45 = 4 tens + 5) to represent numbers (to 1,000).
- MR 1.1 (listed above)
- MR 2.6 Make precise calculations and check the validity of the results from the context of the problem.
Algebra Walk
Changes in Pedagogy

The way we discussed and dissected math concepts provided excellent practice and reflection for doing the same with students.

The way we approached the activities using multimodal strategies helped reinforce the concepts and provided examples for us to bring back to our classrooms.

The MCPT classes have made teaching fun for me and my students.
The way that I have learned to “teach” has transformed “math” into the highlight of my daily experience.

I once looked to my own experience as an elementary math student for inspiration, I now pull from my nights in MCPT.

I have shifted my focus from what I am doing to what they are doing.
These classes have changed the way I present information to my students.

Before I was a procedural teacher - here is a formula, and here are the steps.

Now I try to have students apply their own knowledge to each problem so they can take ownership so they remember it in the future.

Now I use more group activities, have students use tools, measure, draw, make and talk about math problems.
I was moved to 7th grade last year and asked to teach pre-algebra. Many times the standards I had to teach were covered in the MCPT classes shortly before I had to teach that same lesson.

On one occasion, the box-and-whisker plots were taught literally the day before I had to teach it.
Content Linked to Classroom Practice
Positive Outcomes

Increased Confidence Level

Productive Disposition Change

*Productive disposition* refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.

Positive Outcomes for Teachers

• Learning the importance of the use of manipulatives in upper grades
• Adapting the Four Fold way to encourage multiple representations
• Providing opportunities for students to discover ways to solve problems
  • Sense-making approach (SMP #1)
• Recognizing the need to let students struggle
• Focus on ways students think (SPED teacher)
• Becoming more comfortable with content
• Becoming better teachers than ever
Understanding the "why" behind the math
I see math conceptually more than just computationally. I realize that every part of math has a "why" and the instructors of the MCPT classes did a great job making the "why" clear to me.

I realize through taking the MCPT courses that math makes sense. I don't always need to be given a formula (although sometimes this helps) to discover the answer to a problem.
When the opportunity to be part of the DELTA grant came up I was hesitant because I thought math was a pretty black and white subject.

Oh how wrong I was!

I discovered the why behind the math. Looking at the formulas and rules to see the rationale behind them finally clicked and made sense.
PERCENTAGE OF STUDENTS WHO INCREASED ON THE CST MATH 2011 TO 2012 BY MCPT STATUS PROJECT DELTA YEAR 2

<table>
<thead>
<tr>
<th>GROUP</th>
<th>decrease</th>
<th>same</th>
<th>increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparison</td>
<td>30.8</td>
<td>47.4</td>
<td>21.8</td>
</tr>
<tr>
<td>DELTA no MCPT</td>
<td>30.7</td>
<td>46</td>
<td>23.2</td>
</tr>
<tr>
<td>MCPT</td>
<td>18.3</td>
<td>53.2</td>
<td>28.5</td>
</tr>
</tbody>
</table>

n = 13,834

GROUP
n = 1,829

n = 519
Percentage of Students Who Scored Proficient or Advanced on the CST Math 2012 by MCPT Status Project DELTA Year 2

- **Comparison**
  - Proficient or Advanced 2011: 51.3%
  - Proficient or Advanced 2012: 45.6%
  - Total: 18,398

- **DELTA no MCPT Group**
  - Proficient or Advanced 2011: 55.7%
  - Proficient or Advanced 2012: 50.1%
  - Total: 2,204

- **MCPT**
  - Proficient or Advanced 2011: 54.8%
  - Proficient or Advanced 2012: 58.7%
  - Total: 596
Average Percentage Point Increase from Pre to Post on Math Benchmarks 2011-2012

<table>
<thead>
<tr>
<th>Group</th>
<th>Average Percentage Increase from Pre to Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparison</td>
<td>6.9</td>
</tr>
<tr>
<td>DELTA no MCPT</td>
<td>13.6</td>
</tr>
<tr>
<td>MCPT</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Group sizes:
- Comparison: $n = 1,090$
- DELTA no MCPT: $n = 1,609$
- MCPT: $n = 510$
LMT Teacher Content Measure Results
Elementary Number Concepts and Operations
by Group Year 1

<table>
<thead>
<tr>
<th>Group</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparison</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>DELTA no MCPT</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>MCPT</td>
<td>17</td>
<td>21</td>
</tr>
</tbody>
</table>

$n = 9, n = 95, n = 19$
LMT Teacher Content Measure Results
Middle School Number Concepts and Operations
By Group Year 2

- **Comparison**: $n = 23$
  - Pre: 16
  - Post: 17

- **DELTA no MCPT Group**: $n = 61$
  - Pre: 12
  - Post: 16

- **MCPT**: $n = 19$
  - Pre: 12
  - Post: 18
Results – DATA Section

Students of MCPT Teachers:
- show greater improvement on the CST Math,
- are more likely to score proficient/advanced on the CST Math, and
- show greater improvement on the math benchmark
...relative to comparison groups.

MCPT Teachers:
- demonstrate greater improvement on the Elementary Number Concepts and Operations version of the LMT - and -
- demonstrate greater improvement on the Middle School Number Concepts and Operations version of the LMT,
...relative to comparison groups.

Caution should be taken when interpreting results due to small sample size. Despite this limitation, the trends are very promising and would benefit from additional research using a larger cohort.
MCPT prepared Elementary School Teachers who want to teach Middle School

Two teachers moved up to MS for next year

Prepared teachers to pass the CSET

One participant passed after 8 tries prior to MCPT
Challenges

Staying on track with the benchmarks while teaching “MCPT” lessons

Cohort Challenges – Built own culture

Varied Instructor Practices

Attrition due to changes in participant health, family life, or job requirements (e.g. obtaining SPED credential)

Increase in level of rigor
  Some teachers decided they would not use the content at that level
If I had a chance to do this all again…I would have tried out more activities in my classroom …I was too afraid of my administrators getting mad at me for not being on the pacing guide or on task with the other 4th grade teachers. I would take more time for mental math, hands on activities, and physical math games. I have realized that these things are very important for students' learning.
Next Steps

Create materials like these to address the new CCSS standards

Embed the SMPs into the curriculum

Begin more cohorts – FUNDING???
How can we further support participants and administrators to change instruction in preparation for the CCSS?
To Sum It Up

I no longer think of math as simply the name of a class.

The "aha" moments we experienced in these courses were such a thrill. It is as if the instructors have given us permanent x-ray glasses which we can see through all problems.

MCPT has helped me grow as a math learner and as a math teacher.

I will forever be grateful for the experiences during the last three years.

To sum it up in a few words, “What a feeling!”
FOCUS QUESTIONS

• What are the measures of each angle of each pattern block?

• What do you know about the sum of the measures of the interior angles of polygons?

• How does a protractor work?

• When parallel lines are cut by a transversal, what do we know about the angles formed?
THE PATTERN BLOCK PROTRACTOR AND PROPERTIES OF PARALLEL LINES

Participants find the interior angle measures for the six pattern block pieces and the sum of the interior angles of polygons. They then use pattern blocks to construct a protractor, which is used to measure angles and to conjecture about properties of parallel lines.

<table>
<thead>
<tr>
<th>Lesson Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>☐ Explore angle measurements of polygons using pattern blocks</td>
</tr>
<tr>
<td>☐ Increase understanding of angles and protractors</td>
</tr>
<tr>
<td>☐ Understand properties of parallel lines</td>
</tr>
<tr>
<td>☐ Connect lesson goals to standards</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Word Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>☐ protractor</td>
</tr>
<tr>
<td>☐ degrees</td>
</tr>
<tr>
<td>☐ supplementary angles</td>
</tr>
<tr>
<td>☐ complementary angles</td>
</tr>
</tbody>
</table>

<table>
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<th>Focus Questions</th>
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<tr>
<td>☐ What are the measures of each angle of each pattern block?</td>
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<td>☐ How does a protractor work?</td>
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<td>☐ When parallel lines are cut by a transversal, what do we know about the angles formed?</td>
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</table>
SELECTED CA MATH STANDARDS

Grade 4

AF1.3 Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.
AF1.4 Use and interpret formulas (e.g. area = length X width or A = lw) to answer questions about quantities and relationships.
AF1.5 Understand that an equation such as y = 3x + 5 is a prescription for determining the second number when the first number is given.
MG3.1 Identify lines that are parallel and perpendicular.
MG3.2 Identify the radius and diameter of a circle.
MG3.5 Know the definitions of a right angle, acute angle, and obtuse angle. Understand that 90°, 180°, 270°, and 360° are associated respectively with ¼, ½, ¾, and full turns.
MG3.8 Know the definition of different quadrilaterals (e.g. rhombus, square, rectangle, parallelogram, trapezoid).

Grade 5

MG2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles using appropriate tools (e.g., straightedge, ruler, compass, protractor, and drawing software).
MG2.2 Know that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° degrees and use this information to solve problems.

Grade 6

AF 3.1 Use variables in expressions describing geometric quantities (e.g. P = 2w + 2l, A = ½ bh, C = Bd – the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).
AF 3.2 Express in symbolic form simple relationships arising from geometry.
MG 2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.
MG2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving the unknown angles.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern Blocks Angles</td>
<td>MG 3.5, 3.8</td>
<td>MG 2.1, 2.2</td>
<td>MG 2.2</td>
</tr>
<tr>
<td>Polygon Chart</td>
<td>AF 1.3, 1.4, 1.5</td>
<td>MG 2.1, 2.2</td>
<td>AF 3.1, 3.2</td>
</tr>
<tr>
<td>Make a Protractor</td>
<td>MG 3.5</td>
<td>MG 2.1, 2.2</td>
<td>MG 2.2</td>
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<tr>
<td>Parallel Lines</td>
<td>MG 3.1, 3.2</td>
<td>MG 2.1, 2.2</td>
<td>MG 2.1, 2.2</td>
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<tr>
<td>The Regular Hexagon Problem</td>
<td>MG 2.1, 2.2</td>
<td>AF 3.1, 3.2</td>
<td>MG 2.1, 2.2</td>
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<tr>
<td>Two Proofs</td>
<td>MG 2.1, 2.2</td>
<td>MG 2.1, 2.2</td>
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</table>

AF: Algebra and Functions     MG: Measurement and Geometry

UCLA Math Programs for Teachers/LUCIMATH Project
Pattern Blocks Protractors and Properties of Parallel Lines

PPP – PP2
Use a straight edge to draw the following polygons.

- Triangle
- Quadrilateral
- Pentagon
- Hexagon
Use a straight edge to draw the following polygons.

Heptagon

Octagon

Nonagon

Decagon
# POLYGON CHART

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number Of Sides</th>
<th>Number of Diagonals From Any Given Vertex</th>
<th>Number Of “Triangles” Formed</th>
<th>Sum Of Interior Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
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<td>Pentagon</td>
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<td>Hexagon</td>
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<tr>
<td>Octagon</td>
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<tr>
<td>Nonagon</td>
<td>9</td>
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<tr>
<td>Decagon</td>
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<tr>
<td>n-gon</td>
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</table>

1. What pattern did you find for the sum of interior angles of a polygon?

2. Are there any other ways to triangulate the polygons?