Common Core State Standards for Mathematics: Shifts and Implications for Mathematics Instruction

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The Three Shifts in Mathematics

- **Focus** strongly where the standards focus
- **Coherence**: Think across grades and link to major topics within grades
- **Rigor**: Require conceptual understanding, fluency, and application
Shift One: **Focus**

strongly where the Standards focus

- Significantly narrow the scope of content and deepen how time and energy is spent in the math classroom
- Focus deeply only on what is emphasized in the standards, so that students gain strong foundations
Focus in International Comparisons

TIMSS and other international comparisons suggest that the U.S. curriculum is ‘a mile wide and an inch deep.’

“Further evidence of the broad topic coverage characterizing U.S. mathematics curriculum is indicated by the low percentage of TIMSS mathematics topics not included in the U.S. curriculum through grades 4 and 8 compared with the percentage of TIMSS topics that other countries do not include…. On average, the U.S. curriculum omits only 17 percent of the TIMSS grade 4 topics compared with an average omission rate of 40 percent for the 11 comparison countries. The United States covers all but 2 percent of the TIMSS topics through grade 8 compared with a 25 percent noncoverage rate in the other countries. High-scoring Hong Kong’s curriculum omits 48 percent of the TIMSS items through grade 4, and 18 percent through grade 8. Less topic coverage can be associated with higher scores on those topics covered because students have more time to master the content that is taught.”

– Ginsburg et al., 2005
The shape of math in A+ countries

Mathematics topics intended at each grade by at least two-thirds of A+ countries

Mathematics topics intended at each grade by at least two-thirds of 21 U.S. states

### Topic: Number—Numerical Reasoning

<table>
<thead>
<tr>
<th>Item</th>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
</table>
| Maria is thinking of a number. The clues for her number are shown below:<br>• It is a multiple of 5.<br>• It is an even number<br>• It is less than 18.<br>Which of these could be Maria’s number?<br>a. 5  b. 20  c. 8  d. 10 | Fill in the boxes with the correct numbers.<br>\[
\begin{array}{c}
6 \left\lfloor \begin{array}{c}
8 \\
5 \\
4 \\
2
\end{array}
\right. \\
7
\end{array}
\] | \[
\begin{array}{c}
6 \left\lfloor \begin{array}{c}
8 \\
5 \\
2
\end{array}
\right. \\
7
\end{array}
\] |

**NOTE:** This item is from an alternate form of the test and is not included in Exhibits 3 or 6.

<table>
<thead>
<tr>
<th>Item Format</th>
<th>Multiple choice</th>
<th>Short closed constructed-response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational Difficulty</td>
<td>Low (two-digit numbers)</td>
<td>High (three-digit by one-digit long division)</td>
</tr>
</tbody>
</table>
| Cognitive Complexity | Level 2 (requires identifying the number that satisfies three conditions and an understanding of “multiple,” “even,” and “less than”) | Level 3 (requires an in-depth understanding of the division algorithm and significant reasoning skills) \[
(6 \times \ _{a} = \ _{b} \ 82 \text{ and } _{a} \times 6 = 5 
\] |
| Comments | Solution: 17 \times 6 = 102, 27 \times 6 = 162, 37 \times 6 = 222, 47 \times 6 = 282, but 4 \times 6 cannot be 5 \_\_. However, 97 \times 6 is 582 and 9 \times 6 = 54. Alternatively, students might start with the lower box having to be a 4 since there is no remainder and then work backward to fill in the numbers | }
But Much Sharper Focus

### Exhibit 13. Number: Estimation

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brianna bought 4 shirts. Each shirt cost $8.95. Which estimate is closest to the total cost of the shirts that Brianna bought?</td>
<td>The Hong Kong test includes no estimation items.</td>
</tr>
<tr>
<td>a. $32</td>
<td>b. $36</td>
</tr>
</tbody>
</table>

### Exhibit 34. Patterns, Relations and Algebra—Patterns

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Mackey wrote the number pattern below using the rule “subtract 8.” 187, 179, 171, <strong>?</strong>, 165, 147, 139</td>
<td>The Hong Kong test includes no algebra number sentence items.</td>
</tr>
<tr>
<td>What is the missing number in Ms. Mackey’s pattern?</td>
<td></td>
</tr>
<tr>
<td>a. 183</td>
<td>b. 186</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zoey is using bananas and oranges to make the pattern shown below. The rule for her pattern is ABBB. 🍊🍊🍊🍊🍊</td>
<td></td>
</tr>
<tr>
<td>Zoey will follow the rule for her pattern a total of 4 times. How many oranges will Zoey use in all? Show or explain how you got your answer?</td>
<td></td>
</tr>
</tbody>
</table>

### Exhibit 35. Patterns, Relations and Algebra—Number Sentences

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which number sentence is true?</td>
<td></td>
</tr>
<tr>
<td>① 5 + 0 = 5 × 1</td>
<td>① 5 + 0 = 5 × 1</td>
</tr>
<tr>
<td>② 5 + 1 = 5 × 1</td>
<td>② 5 + 1 = 5 × 0</td>
</tr>
<tr>
<td>③ 5 + 0 = 5 × 0</td>
<td>③ 5 + 1 = 5 × 0</td>
</tr>
<tr>
<td>④ 5 + 1 = 5 × 0</td>
<td>④ 5 + 1 = 5 × 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candace wrote the number sentence below. 15 ÷ 3 = □</td>
<td></td>
</tr>
<tr>
<td>Which of these is another way to write Candace’s number sentence?</td>
<td></td>
</tr>
<tr>
<td>A 15 + □ = 3</td>
<td>B 15 ÷ □ = 3</td>
</tr>
<tr>
<td>C 3 + □ = 15</td>
<td>D 3 × □ = 15</td>
</tr>
</tbody>
</table>
Focus in College/Career Readiness
High School: A Major Disconnect

What percentage of mathematics educators reported that their students are prepared for college-level work in mathematics?

- High School Mathematics Instructors: 89%
- Postsecondary Mathematics Instructors: 26%

How the Disconnect Plays Out

• Many students in two-year and four-year colleges need remediation in math
• Remedial classes lower the odds of finishing the degree or program
• Need to set the agenda in high school math to prepare more students for postsecondary education and training
Postsecondary instructors want deeper mastery of fewer things

Postsecondary vs. High school skill ratings
Conley et al., validity study of CCSS

• Just-released survey of over 1,800 postsecondary instructors
• Instructors rated each of the CCSSM content standards in high school as to applicability and importance for college-level work
• Range of courses and institutions

Figures 5 and 6 provide information about the distribution by two- and four-year institutions for the courses as a whole and by content area. Approximately 60% of the courses came from four-year institutions, with the other 40% from two-year institutions. This pattern was fairly consistent for each content area as well, with two exceptions. For the social science courses, the percentage at four-year institutions was slightly higher (56% vs. 34% at two-year institutions). For healthcare courses, the percentage at two-year institutions was higher (56% vs. 45% at four-year institutions).

In order to obtain context for the perceptions of instructors in our sample, we asked several questions about the nature of the courses. Figures 7 through 9 and Table 4 show the demographic information about the courses. Figure 7 shows the level of the course. The survey was intended to capture perceptions of instructors of courses that students encounter at the beginning of their college careers; however, 10% of the respondents considered their
Not all content areas are equally important

Heuristic Importance Rating of CCSSM High School Content Clusters
Not many clusters are important
Bridging the gap: HS Emphases

• The Standards for Mathematical Practice, viewed in connection with mathematical content.

• Modeling and rich applications (see pp. 72, 73 in the standards), which can be integrated into mathematics curriculum, instruction, and assessment.
HS Content Emphases, cont’d

• Number and Quantity: Quantities:
  – Reason quantitatively and use units to solve problems

• Number and Quantity: The Real Number System:
  – Extend the properties of exponents to rational exponents
  – Use properties of rational and irrational numbers

• Algebra: Seeing Structure in Expressions:
  – Interpret the structure of expressions
  – Write expressions in equivalent forms to solve problems

• Algebra: Arithmetic with Polynomials and Rational Expressions:
  – Perform arithmetic operations on polynomials

• Algebra: Creating Equations:
  – Create equations that describe numbers or relationships
HS Content Emphases

• Algebra: Reasoning with Equations and Inequalities:
  – Understand solving equations as a process of reasoning and explain the reasoning
  – Solve equations and inequalities in one variable
  – Represent and solve equations and inequalities graphically

• Functions: Interpreting Functions:
  – Understand the concept of a function and use function notation
  – Analyze functions using different representations
  – Interpret functions that arise in applications in terms of a context

• Functions: Building Functions:
  – Build a function that models a relationship between two quantities

• Geometry: Congruence:
  – Prove geometric theorems

• Statistics and Probability: Interpreting Categorical and Quantitative Data:
  – Summarize, represent, and interpret data on a single count or measurement variable
A milepost, not a finish line

• The college and career ready line is a line best crossed with velocity. In particular, students who wish to pursue STEM majors, or who wish to do college-level work in high school such as AP or IB courses, must progress well beyond the initial threshold of college and career readiness as defined by the standards.
Traditional U.S. Approach

Number and Operations

Measurement and Geometry

Algebra and Functions

Statistics and Probability
Focus on operations within Number and Operations
Shift Two: **Coherence**

Think across grades, and link to major topics within grades

- Carefully connect the learning within and across grades so that students can build new understanding onto foundations built in previous years.

- Begin to count on solid conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.
Coherence example: Progression across grades

“The coherence and sequential nature of mathematics dictate the foundational skills that are necessary for the learning of algebra. The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percents, and negative fractions). The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected.”

<table>
<thead>
<tr>
<th><strong>Gr. 4</strong></th>
<th><strong>Gr. 5</strong></th>
<th><strong>Gr. 6</strong></th>
</tr>
</thead>
</table>
| Addition and subtraction:  
| • Add and subtract like fractions  
| • Add and subtract related fractions (denominators of given fractions should not exceed 12)  
| Multiplication of a proper or improper fraction and a whole number | Addition and subtraction of fractions with unlike denominators:  
| • Add and subtract fractions with unlike denominators  
| Multiplication and division of fractions:  
| • Multiply proper fractions, improper fractions, mixed numbers and whole numbers by proper fractions, improper fractions, and mixed numbers  
| • Divide fractions by whole numbers and whole numbers by fractions | Division of fractions:  
| • Divide proper fractions by proper fractions  
| Mixed calculations with fraction and decimal:  
| • Know how to solve simple calculations with both fractions and decimals |

**Grade 4**

4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

**Grade 5**

5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

**Grade 6**

6.NS. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

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Coherence example: Grade 3

The standards make explicit connections at a single grade

- Multiplication and Division
  - 3.OA.5
- Properties of Operations
  - 3.MD.7a
  - 3.MD.7c
- Area
Unit 1

Angles & Order of Operations
Shift Three: **Rigor**

Equal intensity in conceptual understanding, procedural skill/fluency, and application

- The CCSSM require a balance of:
  - Solid conceptual understanding
  - Procedural skill and fluency
  - Application of skills in problem solving situations

- This requires equal intensity in time, activities, and resources in pursuit of all three
(a) Solid Conceptual Understanding

• Teach more than “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives
• Students are able to see math as more than a set of mnemonics or discrete procedures
• Conceptual understanding supports the other aspects of rigor (fluency and application)
Write four fractions that are all equal to 5: _____, _____, _____, _____

Illustrations of innovative characteristics in the ITN and/or in its appendices are not intended as literal previews of future PARCC assessment tasks. Rather, they are included in the ITN in order to illustrate terms and ideas, signal shifts called for in the standards, and signal PARCC's interest in innovation. They are not
(1 point) Check the box if the given expression is a polynomial.

i. \((3 - t^2)(4 - t^2)(5 - t^2)\) □

ii. \(\pi r^2\) □ polynomial

iii. \(\frac{1}{2}\) □

iv. \(\frac{1}{2x}\) □

v. \(1 - t^2\) □

vi. \(\sqrt{2}\) □
Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
1 hundred + 4 tens = ______

4 tens + 1 hundred = ______

14 tens = ______ tens + ______ tens

= ______ hundred + ______ tens

= ______

90 + 300 + 4 = ______
Hundreds, Tens and Ones

a. 234 = _____ hundreds, _____ tens, _____ ones

b. 809 = _____ hundreds, _____ tens, _____ ones

c. 571 = _____ hundreds, _____ tens, _____ ones

d. 160 = _____ hundreds, _____ tens, _____ ones

e. 67 = _____ hundreds, _____ tens, _____ ones

f. _________ = 3 hundreds, 4 tens, 8 ones

g. _________ = 6 hundreds, 0 tens, 2 ones

h. _________ = 0 hundreds, 0 tens, 5 ones

i. _________ = 0 hundreds, 7 tens, 0 ones

j. _________ = 9 hundreds, 9 tens, 9 ones
Write a number that is greater than $\frac{1}{5}$ and less than $\frac{1}{4}$: ________

Hint: find equivalent fractions for $\frac{1}{5}$ and $\frac{1}{4}$ with denominators 40 or 100.
(b) Fluency

• The standards require speed and accuracy in calculation.

• Teachers structure class time and/or homework time for students to practice core functions such as single-digit multiplication so that they are more able to understand and manipulate more complex concepts.
## Required Fluencies in K-6

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
</tr>
<tr>
<td>6</td>
<td>6.NS.2,3</td>
<td>Multi-digit division</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multi-digit decimal operations</td>
</tr>
</tbody>
</table>
Mark each equation true or false.

☐  $8 \times 9 = 80 - 8$
☐  $54 \div 9 = 24 \div 6$
☐  $7 \times 5 = 25$
☐  $8 \times 3 = 4 \times 6$
☐  $49 \div 7 = 56 \div 8$
If \( A = 356 \times 618 \) and \( B = \frac{2.4}{0.1} \) then what is \( A \div B \) divided by 18?
Fluency in high school

Fluency Recommendations

A/G  Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.1  Fluency in adding, subtracting and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.1b  Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square and other mindful algebraic calculations.
(c) Application

• Students can use appropriate concepts and procedures for application even when not prompted to do so

• Provide opportunities at all grade levels for students to apply math concepts in “real world” situations, recognizing this means different things in K-5, 6-8, and HS

• Teachers in content areas outside of math, particularly science, ensure that students are using grade-level-appropriate math to make meaning of and access science content
On Monday, Joe walked \( \frac{1}{2} \) mile. On Tuesday, Joe walked \( \frac{1}{2} \) mile again. On Wednesday, Joe walked some more. Altogether Joe walked 2 \( \frac{1}{2} \) miles. How far did Joe walk on Wednesday?
A plate of cookies

There were 28 cookies on a plate.
Five children each ate one cookie.
Two children each ate 3 cookies.
One child ate 5 cookies.
The rest of the children each ate two cookies.
Then the plate was empty.

How many children ate two cookies? Show your work.
Suppose Tom wrote check #556 on November 5, 1995, and check #953 on September 26, 1997. What is a good guess for when Tom wrote check #678?
Sale prices

Max bought 2 items that were on sale.

One item was 10% off.

One item was 20% off.

Max says he saved 15% altogether.

a) Could Max be right?

b) Could Max be wrong?
Propane Tanks

Propane tanks are used to store propane gas. Often these tanks are made in the shape of a cylinder with hemispheres on the ends.

The Propane Tank Company makes tanks with this shape, in different sizes. The cylinder part of every tank is exactly 10 feet long, but the radius of the hemispheres, $r$, will be different depending on the size of the tank.

A standard tank measures 6 feet in diameter. The company wants to double the capacity of its standard tank. What should the radius of the new tank be?
It starts with **Focus**

- The current U.S. curriculum is ‘a mile wide and an inch deep.’
- Focus is necessary in order to achieve the rigor set forth in the standards.
- Remember Hong Kong example: more in-depth mastery of a smaller set of things pays off.
## Content Emphases by Cluster

**Grade Four**

**Key:** □ Major Clusters; □ Additional Clusters; ○ Supporting Clusters

<table>
<thead>
<tr>
<th>Operations and Algebraic Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>■ Use the four operations with whole numbers to solve problems.</td>
</tr>
<tr>
<td>○ Gain familiarity with factors and multiples.</td>
</tr>
<tr>
<td>□ Generate and analyze patterns.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number and Operations in Base Ten</th>
</tr>
</thead>
<tbody>
<tr>
<td>■ Generalize place value understanding for multi-digit whole numbers.</td>
</tr>
<tr>
<td>■ Use place value understanding and properties of operations to perform multi-digit arithmetic.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number and Operations--Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>■ Extend understanding of fraction equivalence and ordering.</td>
</tr>
<tr>
<td>■ Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</td>
</tr>
<tr>
<td>□ Understand decimal notation for fractions, and compare decimal fractions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Measurement and Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>○ Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</td>
</tr>
<tr>
<td>○ Represent and interpret data</td>
</tr>
<tr>
<td>□ Geometric measurement: understand concepts of angle and measure angles.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</td>
</tr>
</tbody>
</table>
FAQ 1

• “I’m already pressed for time! How can I address conceptual understanding, procedural skill/fluency, and application?”
FAQ #2

• “You said the shifts were Focus, Coherence, and Rigor. But isn’t what’s important about these standards the Standards for Mathematical Practice?”
FAQ #3

• “Given the standards and tests I’m working with now, won’t focusing hurt my students’ scores?”
The Coming CCSS Assessments Will Focus Strongly on the Major Work of Each Grade

A quality assessment should strive to reinforce focus and coherence at each grade level by testing for proficiency with central and pivotal mathematics rather than covering too many ideas superficially — a key point of the Common Core Standards.

PARCC Releases ITN To Develop Assessments

Design for focus and coherence. Consistent with the design of the CCSSM, the previous iteration of the assessment design adopted by the Leadership Team, and the extended discussion of emphases in the standards in the PARCC Model Content Frameworks, the Mathematics Assessment System as a whole and in each component will focus heavily on the major content of each grade.
Preparing for CCSS assessment—and doing better on today’s assessment

• Remember TIMSS trend—omitting topics on the test was correlated to higher achievement.
• What focus looks like this spring: A tale of two students who score equally
  – One student gets half the points because every question is a coin flip.
  – The other student gets half the points because s/he answers 80% of the major work correctly and 20% of the other work correctly.
  – Both students score the same...but one of them is far better prepared for the next grade.

(The ballpark estimate here is that roughly half the test corresponds to the major work of the grade.)