Using Performance Tasks to Connect Mathematics Instruction to Real-World Problems

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UCLA Curtis Center
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What is a Performance Task?

• A performance task (PT) is an item type designed to provide students with an opportunity to demonstrate their ability to apply their knowledge and higher-order thinking skills to explore and analyze a complex, real-world scenario.

• A mathematics performance task elicits evidence of students’ ability to “bring it all together” to develop a solution plan to the central challenge of the task.
A little history on development of PTs

• First effort (ETS in early specification work)
• Second effort (CTB McGraw-Hill and various subcontractors during pilot phase)
• Third effort (CTB led effort leveraging educators)
• Fourth effort – a collaboration of many
  • Curtis Center at UCLA
  • Callahan Consulting
  • Educators from member states
  • Experts from SCALE, GEARS, NCTM, etc…
Performance Task Blueprint

• Performance Task (60 minutes in duration*)
  – Total of 4-6 items
  – First 1-2 items are independent, machine scorables (for High School they may be hand scored), and allow entry into the task.
  – Final items should be interconnected and cohesive and may be interdependent with each other but not dependent on either of the first two items.
  – Address Claims 2, 3, and 4.
  – *High School PTs may take more than 60 minutes.

• Variants and Scoring Guide
Claims 2, 3, and 4
Performance Tasks focus on Claims 2, 3, and 4

<table>
<thead>
<tr>
<th>Claim/Score Reporting Category</th>
<th>PT Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Concepts and Procedures</td>
<td>0</td>
</tr>
<tr>
<td>2. Problem Solving</td>
<td>1–2</td>
</tr>
<tr>
<td>3. Communicating Reasoning</td>
<td>1-2</td>
</tr>
<tr>
<td>4. Modeling and Data Analysis</td>
<td>2–3</td>
</tr>
<tr>
<td>Total</td>
<td>4-6 items</td>
</tr>
</tbody>
</table>
Smarter Balanced Claims

• **Claim 1: Concepts and Procedures**
  – Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency

• **Claim 2: Problem Solving**
  – Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies

• **Claim 3: Communicating Reasoning**
  – Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others

• **Claim 4: Modeling and Data Analysis**
  – Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems
Claim 2 – Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

A. Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace

B. Select and use tools strategically

C. Interpret results in the context of the situation

D. Identify important quantities in a practical situation and map their relationships.
Claim 3 – Communicating Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

A. Test propositions or conjectures with specific examples.
B. Construct, autonomously, chains of reasoning that justify or refute propositions or conjectures.
C. State logical assumptions being used.
D. Use the technique of breaking an argument into cases.
E. Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.
F. Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
G. Determine conditions under which an argument does and does not apply.
Claim 4 – Modeling and Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

A. Apply mathematics to solve problems arising in everyday life, society, and the workplace.

B. Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

C. State logical assumptions being used.

D. Interpret results in the context of a situation.

E. Analyze the adequacy of and make improvement to an existing model or develop a mathematical model of a real phenomenon.

F. Identify important quantities in a practical situation and map their relationships.

G. Identify, analyze, and synthesize relevant external resources to pose or solve problems.
Performance Tasks...

• Need to be feasible for the school/classroom environment.

• Allow for demonstration of important knowledge and skills, including those that address 21st-century skills, such as critically analyzing and synthesizing information presented in a variety of formats, media, etc.

• Integrate knowledge and skills across multiple Claims and Targets – a key component of college and career readiness.

• Require scoring that focuses on the essence of the Claim(s) and Targets for which the task was written.
Developmentally Appropriate, Engaging, and Accessible

• The Performance Task should be developmentally appropriate.

• The Performance Task may assess mathematics content standards up to the grade level of the target task, but should **primarily** draw upon content standards from earlier grades.

• Items do not need to align to a single standard and may align to a higher level (cluster) or address standards across multiple domains.

• Students of the target age group should have had experience in the context of the task.
Performance Tasks Engagement Features

• **Relevance:** Answers the question why does the educational content *matter* to the student and provides the student with a reason for doing the task.

• **Authenticity:** Requires students to solve real-world problems that have value beyond school.

• **Agency:** Invites students to choose or self-initiate an action.

• **Higher Order Thinking Skills:** Requires students to employ their higher order thinking skills rather than simple recall.

• **Self Assessment:** Permits students to monitor and evaluate their work prior to submission.

• **Overarching Engagement Question:** Asks *why* would a student find this task engaging?
Don't let Smarter assessment rules for PTs limit what you do in the classroom!

• Look for opportunities for students to turn everyday activities into rich performance tasks.
• What issues are relevant for your students?
• What are they learning in science class?
• Don’t constrain your classroom PTs to match the summative blueprint. It’s too restrictive. Just be sure to focus on the practice standards.
**MAT.04.PT.4.ARTP.J.A.155 Claim 4**

<table>
<thead>
<tr>
<th>Sample Item Id:</th>
<th>MAT.04.PT.4.ARTP.J.A.155</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title:</td>
<td>Art Project Fractions</td>
</tr>
<tr>
<td>Grade:</td>
<td>03</td>
</tr>
<tr>
<td>Primary Claim:</td>
<td><strong>Claim 4: Modeling and Data Analysis</strong></td>
</tr>
<tr>
<td></td>
<td>Students can analyze complex, real-world</td>
</tr>
<tr>
<td></td>
<td>scenarios and can construct and use</td>
</tr>
<tr>
<td></td>
<td>mathematical models to interpret and</td>
</tr>
<tr>
<td></td>
<td>solve problems.</td>
</tr>
<tr>
<td>Secondary Claim(S):</td>
<td>Claim 1: Concepts and Procedures</td>
</tr>
<tr>
<td>Primary Content Domain</td>
<td><strong>Numbers and Operations—Fractions</strong></td>
</tr>
<tr>
<td>Secondary Content Domain</td>
<td>Measurement and Data</td>
</tr>
<tr>
<td>Assessment Target(S):</td>
<td>4 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.</td>
</tr>
<tr>
<td></td>
<td>1 F: Extend understanding of fraction equivalence and ordering.</td>
</tr>
<tr>
<td></td>
<td>1G: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</td>
</tr>
<tr>
<td>Standard(S):</td>
<td>4.NF.1, 4.NF.2, 4.NF.3, 4.NF.4, 3.NF.1, 3.NF.2, 3.NF.3, 3.MD.6, 3.MD.7</td>
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<tr>
<td>Mathematical Practice(S):</td>
<td>1, 2, 3, 4, 5, 6</td>
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<tr>
<td>DOK:</td>
<td>3</td>
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<tr>
<td>Item Type:</td>
<td>PT</td>
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<tr>
<td>Score Points:</td>
<td>14</td>
</tr>
<tr>
<td>Difficulty:</td>
<td>M</td>
</tr>
<tr>
<td>How This Task Addresses The “Sufficient Evidence” For This Claim:</td>
<td>The student is given directions to create a design and then asked to construct and use mathematical models within the design to interpret and solve problems.</td>
</tr>
<tr>
<td>Target-Specific Attributes (E.G., Accessibility Issues):</td>
<td>Accommodations may be necessary for students who have fine-motor-skill challenges or are visually challenged.</td>
</tr>
<tr>
<td>Stimulus/Source:</td>
<td>Pattern blocks</td>
</tr>
</tbody>
</table>
Some oldies, but goodies...

<table>
<thead>
<tr>
<th>Notes:</th>
<th>Multi-part performance task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task Overview:</td>
<td>Students are asked to create a design using pattern blocks, figure the unit fraction each block represents if 1 yellow hexagon represents one whole, combine each group of unit fractions, make comparisons among the fractions, and create a design representing a specific value based on what was learned about the shapes and the unit fractions they represent.</td>
</tr>
<tr>
<td>Teacher Preparation/Resource Requirements:</td>
<td>Students should have access to pattern blocks. (Only yellow hexagons, red trapezoids, blue rhombi, and green triangles are needed. Tan rhombi and orange squares are not needed.) <a href="http://www.aug.edu/~lcrawford/Tools/pattern_blocks.pdf">http://www.aug.edu/~lcrawford/Tools/pattern_blocks.pdf</a> has cutout pages of pattern blocks. These will need to be prepared in advance. Each student should get a minimum of 1 yellow hexagon, 2 red trapezoids, 3 blue rhombi, and 6 green triangles. Plain white paper or 1-inch isometric dot paper is also needed. <a href="http://www.printablepaper.net/preview/IsometricDots-1inch">http://www.printablepaper.net/preview/IsometricDots-1inch</a> has a free downloadable version of the isometric dot paper.</td>
</tr>
<tr>
<td>Teacher Responsibilities During Administration:</td>
<td>Monitor individual work; hand out and collect materials as required for each session.</td>
</tr>
<tr>
<td>Time Requirements:</td>
<td>Two sessions totaling no more than 120 minutes. Each part of this task should be done in sequential order.</td>
</tr>
</tbody>
</table>
The Task:

Session 1

Art Project with Pattern Blocks

Use the pattern blocks that your teacher gives you to help you with this task. There are five parts to this task, and you must work through them in order.

Be sure you have the following pattern blocks before you begin.

Yellow Hexagon  Red Trapezoid  Blue Rhombus  Green Triangle
Part A

Each yellow hexagon pattern block represents one whole. The other colored pattern blocks represent a fraction of the whole yellow hexagon.

Write the fraction of the yellow hexagon each colored shape represents.

1 yellow hexagon represents ______ yellow hexagon.

1 red trapezoid represents ______ yellow hexagon.

1 blue rhombus represents ______ yellow hexagon.

1 green triangle represents ______ yellow hexagon.
Grade 4 Sample PT ~ Art Project with Pattern Blocks

Part B

Use the pattern block shapes to make a picture for an art project.

These rules for making your picture must be followed:

- At least one of each shape must be included in the picture.
- The shapes may touch the edges of other shapes.
- The shapes must not overlap each other.

Click on a shape and then click in the space below to put a shape in the picture. Continue as many times as necessary.

Yellow Hexagon  Red Trapezoid  Blue Rhombus  Green Triangle
Grade 4 Sample PT ~ Art Project with Pattern Blocks

Part C

Count all the colored shapes you used in your picture. Write the total number of each colored shape you used in the picture.

_____ yellow hexagon(s)  _____ blue rhombus (rhombi)

_____ red trapezoid(s)  _____ green triangle(s)

When all of the shapes of one color are combined, they represent a fraction or a mixed number of yellow hexagons. Complete the sentences below showing these fractions or mixed numbers.

_____ red trapezoid(s) represents _____ yellow hexagon(s).

_____ blue rhombus (rhombi) represents _____ yellow hexagon(s).

_____ green triangle(s) represents _____ yellow hexagon(s).

This is the end of Session 1. You will not be able to go back to Parts A, B, or C once you click “Submit.”
For his art project, Carter used 2 yellow hexagons, 5 red trapezoids, 2 blue rhombi, and 7 green triangles. A picture of Carter’s art project is shown below.
Grade 4 Sample PT ~ Art Project with Pattern Blocks

Carter thinks that he can use 7 green triangles and 2 blue rhombi to show that \( \frac{7}{6} \) is greater than \( \frac{2}{3} \) \( \left( \frac{7}{6} > \frac{2}{3} \right) \). Use pictures, numbers, and/or words in the space below to show whether Carter’s thinking is correct or not.

Carter also thinks he can use 2 yellow hexagons and 2 blue rhombi to show that \( \frac{2}{1} \) is equal to \( \frac{2}{3} \) \( \left( \frac{2}{1} = \frac{2}{3} \right) \).

- He says that the sizes of the shapes do not matter.
- He counts the number of each shape and uses this number as the numerator in each fraction.
- He says that the fractions are equal because the numerators are equal.

Use pictures, numbers, and/or words in the space below to show whether Carter’s thinking is correct or not.
Use the pattern block shapes to make a new picture. This picture must represent a value that is equal to \(5\frac{1}{6}\) yellow hexagons.

The other rules still apply:

- At least one of each shape must be included in the picture.
- The shapes may touch the edges of other shapes.
- The shapes must not overlap each other.

Click on a shape and then click in the space below to put a shape in the picture. Continue as many times as necessary.
Grade 4 Sample PT ~ Art Project with Pattern Blocks

Explain how you know the shapes in your picture represent $5\frac{1}{6}$ yellow hexagons.

End of Session 2
Sample Top-Score Response:

Part A
1 red trapezoid represents $\frac{1}{2}$
1 blue rhombus represents $\frac{1}{3}$
1 green triangle represents $\frac{1}{6}$

Part B
Not scored. The following illustration is an example top-score response only. Whatever the student constructs must be interpreted for Part C.

Part C
5 red trapezoids is the same as $\frac{5}{2}$ (or $2 \frac{1}{2}$)
8 blue rhombi is the same as $\frac{8}{3}$ (or $2 \frac{2}{3}$)
7 green triangles is the same as $\frac{7}{6}$ (or $1 \frac{1}{6}$)
**Part D**

I can show that Carter’s thinking is correct and that \( \frac{7}{6} > \frac{2}{3} \). It takes 6 triangles to make 1 whole hexagon. That leaves one extra or \( \frac{1}{6} \). It takes 3 rhombi to make 1 whole hexagon. I have 2 rhombi, which is \( \frac{2}{3} \). I know that \( \frac{2}{3} \) is less than 1 whole, so it also has to be less than \( \frac{7}{6} \).

I can show that Carter’s thinking is not correct and that \( \frac{2}{1} \) is not equal to \( \frac{2}{3} \). The hexagons represent wholes, and the fraction \( \frac{2}{1} \) means that there are 2 wholes. It would take 3 rhombi to make 1 whole. Carter is using 2 of them, which is less than 1 whole. That means Carter is wrong when he says 2 wholes is equal to \( \frac{2}{3} \). Carter would need \( \frac{6}{3} \) or 6 rhombi to equal 2 whole hexagons.

**Part E**

Creates a new picture that uses at least one of each colored shape with no overlap that represents \( 5\frac{1}{6} \) yellow hexagons. One example is shown below; however, there are many ways to correctly complete this part.

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**Example:** “I know my picture equals \( 5\frac{1}{6} \) hexagons because it takes 2 trapezoids to make 1 hexagon, 3 rhombi to make 1 hexagon, and 6 triangles to make 1 hexagon. I started with 2 hexagons. Then I used 4 trapezoids, which equals 2 more hexagons. It takes 2 triangles to make 1 rhombus, so 2 triangles + 2 rhombi = 1 hexagon. That makes 5 hexagons. The remaining triangle is \( \frac{1}{6} \) of a hexagon, so I have the same as \( 5\frac{1}{6} \) hexagons in my picture.”
## Sample Item ID:
MAT.08.PT.4.MYPET.A.415

## Title:
Cost of a Pet

## Grade:
08

### Primary Claim:
**Claim 4: Modeling and Data Analysis**
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

### Secondary Claim(s):
- **Claim 1: Concepts and Procedures**
  Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
- **Claim 2: Problem Solving**
  Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

### Primary Content Domain:
Statistics and Probability

### Secondary Content Domain(s):
Functions

### Standard(s):
8.EE.6, 8.F.4, 7.EE.3, 6.RP.2, 5.NBT.7

### Mathematical Practice(s):
1, 2, 4, 5, 6

### DOK:
4

### Item Type:
PT

### Score Points:
11

### Difficulty:
M

### How this task addresses the "sufficient evidence" for this claim:
The student must use information derived from research to estimate the costs to adopt and maintain a pet. This work will be supported by the use of calculations, graphical representation of data, and generalizations using algebra.

### Target-specific attributes (e.g., accessibility issues):
Accommodations may be necessary for students who have visual challenges. Technology is needed to enable students to create a circle graph, bar graph, or line graph.

### Stimulus/Source:
[www.petsmart.com](http://www.petsmart.com)
Pre-work: In groups or as a whole class, students brainstorm what items are needed to maintain a pet over time. [Students brainstorm ideas for items that are needed to maintain a pet over time. At the end of a class discussion, the teacher should elicit from students a list of items, possibly including food, toys, a cage/carrier/bed, a leash or collar, veterinary visits, litter box. The teacher should note that not all of these items are needed for all types of animals.]

Day 1: With partners, students decide which type of pet they want to adopt. Students use a set of provided “Web sites” to look up the costs of necessary items for the chosen pet.

Day 2: Students individually estimate the cost of adopting and maintaining their chosen pet for 1 year. Students explain why their estimate is reasonable. As part of the explanation, the student must make and refer to a line graph showing the monthly increase in money spent over the year.

Teacher preparation: The day before the administration of the task, there is pre-work that is designed to encourage students to begin thinking about the upcoming task. The pre-work involves a class brainstorming session and discussion. The remainder of the task involves a group assignment for one day followed by scored portions of the task on the second day. During Session 1, students work with partners to develop a list of items that are needed to own a pet and the costs of those items. During Session 2, students work individually to estimate the cost of raising a pet for the first and second year and the relationship between time and cost. Students provide justifications for their answers.
Teacher Preparation: Students will be grouped in partners or small groups. If possible, a student who owns or has owned a pet should be placed in each group.

Teacher says: Today you are going to do some research about the cost of owning a pet. You will create a list of items that you will need for your pet and the cost of each item. You will need this list and associated costs in order to complete the upcoming performance task.

First, choose the type of pet that you [by partner or group] would like to research. The choices are a dog, a cat, a hamster, or a parakeet.

These links will direct you to Web pages that might be useful to you: [Note that all of the Web sites below are fictitious, and would need to be developed. A PowerPoint file shows some examples of what the Web sites would look like as part of the fictitious cached file available with this task.]

PlusmaxPetSupplies.com [sells pet supplies only; gives info about how much and how often to feed pets]
AndreasPetShop.com [sells pets, and some (but not all) supplies]
PetFacts.info [gives info on what items you need to have a pet]
MeAndMyPets.com [a personal blog about someone who has many pets; this site will not contain any useful information for students]
AnimalRescueGroup.org [students can use this site to choose to adopt a pet instead of buy one]
DowntownVeterinaryCare.com [a veterinarian’s site that will give estimates on how often a pet should go to the veterinarian and what a typical visit might cost]
HappyDayPettingZoo.com [another site without any useful info; this site will have the hours and prices for visiting a local petting zoo]
Welcome to Plusmax Pet Supplies!
Click on the tabs on the left to browse our inventory.

Our Hours
Monday to Saturday 10am-9pm
Sunday 10am-6pm

Shop Now:
• Dog
• Cat
• Bird
• Fish
• Reptile
• Small Pets (hamsters/gerbils/mice)
  - Bedding
  - Water bottles
  - Cages and habitats
  - Food
  - Toys

Search our site:
We carry three different small pet cages in our store:

- **Great Choice™ Pet Home for Hamsters**
  - Rating: ★★★★★
  - Our Price: $19.99

- **Critter Universe ECO**
  - Rating: ★★★★★
  - Our Price: $21.99

- **All Living Things® Hamster Starter Kit**
  - Rating: ★★★★★
  - Price: was $26.99
  - now: $22.94

Shop Now:

- Dog
- Cat
- Bird
- Fish
- Reptile
- Small Pets (hamsters/gerbils/mice)
  - Bedding
  - Water bottles
  - Cages and habitats
  - Food
  - Toys
Welcome to Andrea’s!
Click on the animal names above to see what pets we have in our store.
<table>
<thead>
<tr>
<th>Cats</th>
<th>Dogs</th>
<th>Hamsters</th>
<th>Parakeets</th>
<th>Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Russian Dwarf Hamsters" /></td>
<td><img src="image2" alt="Chinese Dwarf Hamster" /></td>
<td><img src="image3" alt="Fancy Russian Dwarf Hamster" /></td>
<td><img src="image4" alt="Winter White Hamster" /></td>
<td><img src="image5" alt="Robo Dwarf Hamster" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Our Price:</strong> $15.99</td>
<td><strong>Our Price:</strong> $15.99</td>
<td><strong>Our Price:</strong> $15.99</td>
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</tr>
</tbody>
</table>
Create a list of items you will need for your pet and the prices of the items.

Cost of Owning and Caring for a Pet

Part A
Enter the list of pet items and costs you created yesterday, including the initial cost to buy or adopt your pet.

Based on your research, estimate the cost of adopting and maintaining a pet <dog, cat, hamster, parakeet> for one year.

You may click on the links to the Web pages you looked at yesterday to help you revise or adjust your list in order to determine the cost for an entire year.

Part B
What do you estimate the cost of adopting and maintaining a pet <dog, cat, hamster, parakeet> for one year is?

$ ___________________

Provide a thorough explanation below that proves you have thought of all the costs associated with owning your choice of pet. Be sure to account for all one-time costs, as well as recurring costs and the frequency of those costs. Adjust your total above if needed.
Next, support your answer by using the line graph tool below to create a line graph that shows the months of the year and the total amount of money you have spent on your pet through the end of each month. Your graph should show how the amount of money you have spent increases over time.

Data Points:

Click to add a data point
Grade 8 Sample PT ~ Cost of Owning & Caring for a Pet

**Part C**

Estimate the cost of maintaining your pet <dog, cat, hamster, parakeet> for *one additional year* after the first.

$ ___________________

Again, provide a thorough explanation below that proves you have thought of all the costs associated with owning your choice of pet for one *additional year*.

**Part D**

Write a linear equation that best fits the total cost of adopting and maintaining your pet for a given number of *months*. Assume that your pet will not need to go to the veterinarian throughout this time.

Equation: ________________________________

Explain how you created your model equation. Be sure to describe what the variables represent and how you determined the slope and intercept for your equation.
Sample Top-Score Response:
[Note: This response is based on a student choosing a hamster.]

Group work:
My group researched the cost of owning a hamster. (Group work not scored.)

Part A
Items needed:
- Carefresh Basic Pet Bedding (14-liter bag) $5.99
- All Living Things Water Bottles for Small Animals $3.99
- All Living Things Hamster Starter Kit $22.94
- Great Choice Hamster & Gerbil Food (5 pounds) $6.99
- Super Pet Small Animal Run-About Ball $7.99
- Super Pet CritterTrail Puzzle Playgrounds $8.99
- Winter White Hamster $15.99
The cost of adopting and maintaining a pet hamster for one year is $231.62.

**Total Amount of Money Spent on Hamster after Each Month**

**Explanation:**
I know that my estimation is accurate because it includes the costs of all items that I would need to adopt and maintain a pet hamster for one year (assuming that my hamster does not need to go to the veterinarian). I looked on a pet supply Web site and on a pet shop Web site in order to find the costs of the items needed.
Going Green

The “Go Green” company held a week-long recycling contest at your school. The grade that collects the most bottles and cans wins the contest.

Your task is to determine which grade won the “Go Green” contest.

The items that can be recycled are shown.

- Plastic bottles
- Aluminum cans
- Glass bottles

Students from the 3rd grade, 4th grade, and 5th grade classes collected bottles and cans and brought them to school.

Table 1 shows how many bottles and cans each grade collected on each day of the week.

<table>
<thead>
<tr>
<th>Class</th>
<th>Bottle and Cans Collected Each Day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Monday</td>
</tr>
<tr>
<td>3rd Grade</td>
<td>50</td>
</tr>
<tr>
<td>4th Grade</td>
<td>70</td>
</tr>
<tr>
<td>5th Grade</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 1. Bottles and Cans Collected
1

Use **Table 1** to help you answer this question.

On which days did the 3rd grade class collect the most cans and bottles? Select **all** that apply.

- ☐ Monday
- ☐ Tuesday
- ☐ Wednesday
- ☐ Thursday
- ☐ Friday

<table>
<thead>
<tr>
<th>Item</th>
<th>Claim</th>
<th>Domain</th>
<th>Target</th>
<th>DOK</th>
<th>Content</th>
<th>MP</th>
<th>Key(s)</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>2</td>
<td>NBT</td>
<td>2D</td>
<td>1</td>
<td>1.NBT.B.3</td>
<td>1</td>
<td>D and E</td>
<td>1</td>
</tr>
</tbody>
</table>

**Rubric** (1 point): The student selects both Thursday and Friday.

<table>
<thead>
<tr>
<th>Item</th>
<th>Claim</th>
<th>Domain</th>
<th>Target</th>
<th>DOK</th>
<th>Content</th>
<th>MP</th>
<th>Key(s)</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2</td>
<td>2</td>
<td>NBT</td>
<td>2D</td>
<td>1</td>
<td>1.NBT.B.3</td>
<td>1</td>
<td>30</td>
<td>1</td>
</tr>
</tbody>
</table>

**Rubric** (1 point): The student enters 30.

2

Use **Table 1** to help you answer this question.

On Tuesday, how many more bottles and cans did the 4th grade class collect than the 3rd grade class?
Commentary: The purpose of these first two questions is primarily to assess whether the student (1) understands the context and the representation (a table) and (2) can identify and infer relevant quantities and perform typical calculations (compare magnitudes).

The context is reasonably authentic. Many schools have recycling contests to encourage students to bring in bottles and cans to recycle. Keeping track of how many each class brings in each day is a good experience for students to record and represent quantities. This task is about deciding which class wins the school recycling contest.

Rationale for Content:
The content is securely held, being essentially Grade One: 1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
Since these questions are intended to be accessible for most students, the numbers have been kept simple so as not to create arithmetic computational obstacles in obtaining evidence of Claims 2, 3, or 4.
Rationale for Claim:
The fact that the student must extract the quantities, choose the procedure, and calculate the answer from a well-posed problem is what makes both items 1 and 2 align to Target 2D:
Claim 2, Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Rationale for DOK:
Both items are straightforward DOK 1: From the Cognitive Rigor Matrix/Depth of Knowledge chart,
-Retrieve information from a table or graph to answer a question.
You are the contest judge. You need to figure out who won the contest. Did 3rd grade, 4th grade, or 5th grade win the “Go Green” contest?

Use words and numbers to clearly explain:
- which grade won the contest, and
- how you know they collected the most cans and bottles.

<table>
<thead>
<tr>
<th>Item</th>
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<th>Target</th>
<th>DOK</th>
<th>Content</th>
<th>MP</th>
<th>Key(s)</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
<td>3</td>
<td>NBT</td>
<td>3F</td>
<td>3</td>
<td>2.NBT.B</td>
<td>3</td>
<td>See rubric</td>
<td>2</td>
</tr>
</tbody>
</table>

**Rubric:**
2 points: The student develops an approach (add the totals and compares) to determine the winner AND gives a justification for the result.
1 point: The student correctly states that 3rd grade won but without an explanation as to why OR gives correct calculations but with a logically flawed justification.
0 points: All other responses
Commentary: This question is designed to provide 3rd grade students an opportunity to develop and justify a result. It is reasonably authentic and addresses the overarching question which is — who won the contest? The key elements are that students are not told what quantities to use, how they are operated on, nor what to conclude from them. This increases the depth of knowledge. This question is not meant to be overly difficult in terms of numeric computational complexity, lest it distract students from the communicating reasoning aspect of the question.

Rationale for Content:
The content is securely held, second grade NBT:
2.NBT.B Use place value understanding and properties of operations to add and subtract.
5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Again, the numbers are designed to be easy to work with so as not to be an obstacle for students focusing on communicating reasoning.
Rationale for Claim:
The work required for this problem has a few necessary steps: (1) Students must develop a chain of reasoning, i.e., interpret the context (winning class collects the most for the week). This requires adding up the amounts collected by each class each day and then comparing the totals.
NOTE: By design, this problem does not tell the students to add up the totals and compare. This takes away the scaffolding and requires the student to select the quantities and appropriate operation to use on them, based on the context. (2) Students must provide an explanation to justify a claim. Correctly adding the amounts does not provide a logical argument. The student must connect the computation to the reasoning, essentially, that the 3rd grade total is greater than the totals for the 4th and 5th grade classes, hence they win the contest.
Claim 3, Target B: Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but should do so with a lesser degree of scaffolding than tasks that assess Target A alone.

Rationale for DOK:
This question qualifies for DOK 3 for a few reasons. From the Cognitive Rigor Matrix/Depth of Knowledge chart,
ANALYZE (DOK3)
-Compare information within or across data sets or texts
-Analyze and draw conclusions from data, citing evidence
EVALUATE (DOK3)
-Cite evidence and develop a logical argument
What follows are sample responses and scoring annotations for Item 3.

3rd grade won because I added up each one and got the most for 3rd grade. 3rd grade had 410, 4th grade has 390, and 5th grade had 400.

5th grade is 80+80+80+80+80=400 total
4th grade is 70+90+100+50+80=390 total
3rd grade is 50+60+90+120+90=410 total

The student correctly calculated the total number of bottles and cans collected by each grade, as well as explained why 3rd grade was the winner of the contest. The response contains evidence of the student’s competence in reasoning to the full extent that these processes apply to this item.
3rd grade won the "Go Green" contest. I know that because I added 120+90+90+60+50 and got 410

4th grade got 290 bottles and cans.

5th grade got 400 bottles and cans.

3
120
90
90
60
+50

410

The student *incorrectly* calculated the total number of bottles and cans collected by each grade (error in 4th grade), but *correctly* determined the winner based on those calculations and provided a logical explanation as to why 3rd grade won the contest. The response contains evidence of the student’s competence in reasoning and while it contains a minor flaw, it does not detract from a demonstration of full understanding.
3rd grade 410
4th grade 390
5th grade 400

- 3rd grade won the "Go Green" contest
- I know that 3rd grade won because I count how many bottles and cans they have each day.

SCORE POINT 1

The student correctly calculated the total number of bottles and cans collected by each grade, as well as correctly identified the winner of the contest. However, “because I count how many bottles and cans they have each day” is not the reason the 3rd grade class won the contest. The student was unable to make the logical connection and has demonstrated only a partial understanding of the mathematical content and practices essential to this item.
<table>
<thead>
<tr>
<th>Grade</th>
<th>Count 1</th>
<th>Count 2</th>
<th>Count 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3rd</td>
<td>50</td>
<td>60</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td></td>
<td>+90</td>
</tr>
<tr>
<td>4th</td>
<td>70</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td></td>
<td>+80</td>
</tr>
<tr>
<td>5th</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>+80</td>
</tr>
</tbody>
</table>

3rd grade: 410 bottles and cans
4th grade: 390 bottles and cans
5th grade: 400 bottles and cans

3rd grade won the "Go Green" contest because they have 410 bottles and cans.

SCORE POINT 1

The student correctly calculated the total number of bottles and cans collected by each grade as well as correctly identified the winner of the contest. However, “and 4th and 5th did not” is not a valid reason for why the 3rd grade class won the contest. The student was unable to make the logical connection and has demonstrated only a partial understanding of the mathematical content and practices essential to this item.
Under Development...new Practice Test PTs

Although the student correctly calculated the total number of bottles and cans collected by each grade, he/she did not identify the winner of the contest, nor justify why that grade won. No evidence is present that demonstrates the student’s competence in reasoning essential to this item.
The student made two errors. First, he/she did not add the total number of bottles and cans correctly. Second, based on the incorrect calculations, he/she did not draw a correct conclusion. Based on the addition shown in the response, 5th grade should have won the contest but this student claimed that 4th grade won. No evidence is present that demonstrates the student’s competence in reasoning essential to this item.
The 2nd grade class got excited about the "Go Green" contest and wanted to join in. They started collecting bottles and cans on Wednesday, even though they missed the first two days.

Complete the table to show a way for the 2nd grade class to still win the contest.

<table>
<thead>
<tr>
<th></th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2nd Grade</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Item</th>
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<th>Domain</th>
<th>Target</th>
<th>DOK</th>
<th>Content</th>
<th>MP</th>
<th>Key(s)</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>#4</td>
<td>4</td>
<td>OA</td>
<td>4A</td>
<td>2</td>
<td>2.OA.A.1</td>
<td>2,4</td>
<td>See rubric</td>
<td>1</td>
</tr>
</tbody>
</table>

Rubric:
1 point: The student enters any numbers in the three days that add up to more than 410 (the total from 3rd grade for the week). NOTE: Do not grade on "reasonableness", i.e., answers like 1,000,000 are allowed.
0 points: All other responses

One example of a correct response:

<table>
<thead>
<tr>
<th></th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2nd Grade</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>
Commentary: The purpose of the question is to increase the level of abstraction for a 3rd grade student to solve a reasonably authentic "extension" question, involving hypothetical 2nd grader students wanting to join the contest late. The student must be able to reason that, even though they have less time, the 2nd grade class could still win if they collect more than the 3rd grade class. This question splits off the modeling from the explanation (See Question #3).

Rationale for Content: The content is securely held, it moves on the progression from Grade 2 Operations and Algebraic Thinking:

2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions

Note that the numbers exceed the "within 100" description, but 3rd grade students should, by the time of the PT, be comfortable with numbers with values of several hundred.
**Rationale for Claim:**

This question is different in that students are *not given specific numbers to work with*. This increases the abstraction. They need to interpret the context and invent numbers that will reasonably model a hypothetical scenario. In some sense, this is a *design under constraints* type of modeling question. That is what makes this a Claim 4: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

Claim 4, Target A: Problems used to assess this target for Claim #4 should not be completely formulated (as they are for the same target in Claim #2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

**Rationale for DOK:**

This question is DOK 3 for the following reasons. From the *Cognitive Rigor Matrix/Depth of Knowledge* chart,

UNDERSTAND (DOK 3)
- Use concepts to solve non-routine problems
- Explain reasoning when more than one response is possible

CREATE (DOK 3)
- Develop an alternative solution
Use your answers from questions 3 and 4 to explain how the 2nd grade class could have won the “Go Green” contest.

<table>
<thead>
<tr>
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<th>Target</th>
<th>DOK</th>
<th>Content</th>
<th>MP</th>
<th>Key(s)</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>#5</td>
<td>3</td>
<td>OA</td>
<td>3A</td>
<td>3</td>
<td>2.OA.A</td>
<td>3</td>
<td>See rubric</td>
<td>2</td>
</tr>
</tbody>
</table>

**Rubric:**
2 points: The student takes the value from Question 4 and states that it is more than the winning number from Question 3 (e.g., 410). NOTE: For full credit the student cannot just give a number (e.g., 800) and state, "it is more". We are looking for a logical statement that explicitly makes use of the information.
1 point: The student states the information but does not fully make the logical connections.
0 points: All other responses

NOTE: We anticipate people will disagree with some of these samples. We want to highlight that we are not looking for just evidence of understanding, but we are looking for evidence of communicating reasoning. In piloting, many 3rd grade students understood and correctly answered the question, but did not communicate or justify the result at a level we want as the expectation for Grade 3.
Under Development...new Practice Test PTs

Commentary: This question is designed to provide an opportunity for students to create an explanation based on several pieces of information. Students cannot just make calculations or statements without justification.

Rationale for Content:
The content is securely held, it moves on the progression from Grade 2 Operations and Algebraic Thinking:
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions

Note that the numbers exceed the "within 100" description, but 3rd grade students should, by the time of the PT, be comfortable with numbers with values of several hundred.
**Rationale for Claim:**
This question addresses Claim 3. Students must put together several pieces of information to support a claim.

Claim 3, Target A: Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture.

This question is DOK 3 for the following reasons. From the *Cognitive Rigor Matrix/Depth of Knowledge* chart,

**UNDERSTAND (DOK 3)**
- Use concepts to solve non-routine problems
- Explain reasoning when more than one response is possible

This is non-routine problem, with many different possible answers. The key aspect of the argument must be that the numbers proposed for 2nd graders in Question #4 must be explicitly compared to the totals in Question #3, and the logical connection is that that the amount is more, and whoever collects the most wins the contest.
2nd grade could have won the contest if they collected 260 bottle and cans on Wednesday, 100 on Thursday, 300 on Friday. If you add it up it will beat 3rd grade.

\[
\begin{align*}
300 \\
+260 \\
+100 \\
\hline
660 \\
\end{align*}
\]

Bottles and cans

The student selected values for bottles and cans collected that exceeded 410 (more than 3rd grade) and he/she explained why a total of 660 bottles and cans would result in the 2nd grade class winning the contest. The student has demonstrated a **full and complete** understanding of all mathematical content and practices essential to this item.
2nd grade could have won the contest if they collected 120 on Wednesday and 200 on Thursday and 100 on Friday and that would bet 3rd grade had 410 and if you add 120, 200, 100, it would be 420 and 420 is greater than 410.

The student selected values for bottles and cans collected that exceeded 410 (more than 3rd grade) and he/she explained why a total of 420 bottles and cans would result in the 2nd grade class winning the contest. The student has demonstrated a full and complete understanding of all mathematical content and practices essential to this item.
Well, I put 500 because 3rd grade had 410. So if I put 500 then they will have more.

The student selected values for bottles and cans collected that exceeded 410 (more than 3rd grade) and he/she did not explain why it is important for 500 to more than 410. The student’s response contains some of the attributes of an appropriate response but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task.
2nd could have won the contest if they collected 200 cans
Wednesday - Friday.

\[
\begin{array}{c|c}
136 & 200 \\
136 & 200 \\
136 & 200 \\
408 & 410 \\
\hline
600 & 600 \\
\end{array}
\]

600 is greater than 410

The student selected values for bottles and cans collected that exceeded 410 (more than 3rd grade) and he/she did not explain why it is important for 600 to more than 410. The student’s response contains some of the attributes of an appropriate response but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task.
Well they won because in number 3 there says that they have to win the contest because they wanted to participate in the "Go Green" so I got from that answer 800.

Although the student selected values for bottles and cans collected that exceeded 410 (more than 3rd grade), he/she did not explain why the value of 800 bottles and cans would be a way for 2nd grade to win the contest. There are significant omissions or irregularities that indicate a lack of comprehension in regard to the mathematical content and practices essential to this task.
General Ideas for Writing Items/Tasks

- Brainstorm task ideas and stimuli with at least one other person before beginning the development process.
- Make a list of the activities you or other family members do during the day that can be turned into an item or performance task.
- Look for brochures on interesting landmarks, facts about the area, etc. found in nearly every hotel and city visitor center.
- Start with the idea for the task, then look for a way to weave it into a Claim 2, 3 or Claim 4 target, as well as the content domain and CCSS cluster.
- Again ask yourself, “Why would a student of this age/grade level care about the task/topic?”
General Ideas for Writing Items/Tasks

– Judy’s list of ideas from a 12-hour period of time:

• Banquet planning – between 30 - 40 participants (can’t confirm until that evening), pay for buffet ($35 per person charged by number of people that attend) or guarantee 40 dinners at $33 per person.

• Should I “bundle” internet/phone/cable service?

• Gas company (propane) just instituted a yearly rental charge on my propane tank, in addition to monthly service charge and per gallon charges for filling tank during the winter months. Is it time to change over to natural gas? (Means buying a new fireplace insert that uses natural gas instead of propane and paying the one-time hook-up charge for the natural gas line.)

• Need to have the tile professionally cleaned, company charges by the square foot plus service call. I know the size of tiles (16 in. squares), but the grout makes it extra and I have irregular areas throughout the house with tile.

• Need to “winterize” my flower/plant beds with pine straw. One bale can cover 3 square meters. The landscaping plan shows lots of irregular shaped beds in my yard, how many bales do I need to purchase?
Questions?