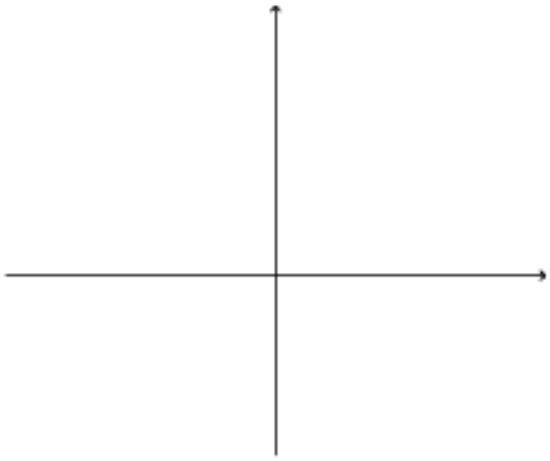
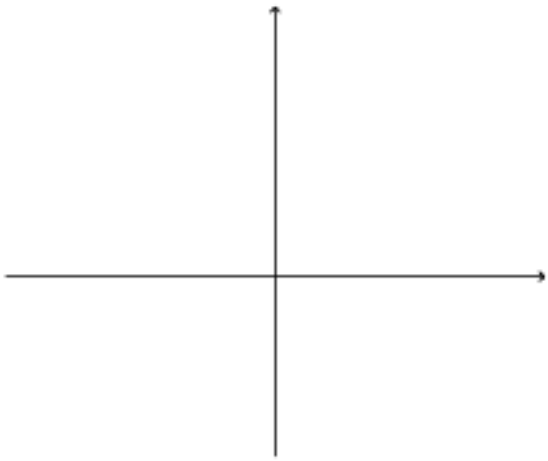
In this set of problems, you will examine how parabolas behave under different transformations of the coordinate plane. You will then use these transformations to gain a graphical understanding of completing the square.

1. Consider the parabola P defined by $y = x^2$.

   a. What is the equation of the parabola obtained by translating P two units to the right? Sketch and label both parabolas below. Make sure to label your axes appropriately.



   b. What is the equation of the parabola obtained by translating P three units up? Sketch and label both parabolas below. Make sure to label to label your axes appropriately.



2. a. Explain how to transform the parabola given by the graph of $y = x^2$ into the parabola given by the graph of $y = x^2 - 4x - 4$ using translations.

b. Use part a) to help you find the roots of $y = x^2 - 4x - 4$.

3. In this question we study how a parabola behaves under a dilation.
   a. If you dilate the graph of $y = x^2$ with scale factor 3 and center of dilation (0,0) do you still get a parabola? Explain.

   b. If you dilate the graph of $y = x^2$ with scale factor $s > 0$ and center of dilation (0,0), do you still get a parabola? Explain.

c. If you dilate the graph of $y = ax^2 + bx + c$ with scale factor $s > 0$ and center of dilation (0,0) do you always get a parabola? Explain.

d. Explain why all parabolas in the plane (defined by quadratic equations of the form $y = ax^2 + bx + c$) are similar.

For the Professional

1. Below are some sequences of transformations that are applied to the parabola given by the graph of $y = x^2$. For each one, write an equation for the new parabola.

   a. Translate left 2 and then dilate with center (0,0) by a factor of 3.

   b. Translate up 1 and then dilate with center (0,0) by a factor of 3.

   c. Dilate with center (0,0) by a factor of 3 and then translate up 1.

   d. Translate left 2, translate up 1, and then dilate with center (0,0) by a factor of 3.

2. Below are some equations of parabolas. In each case, describe a series of transformations that will take the graph of $y = x^2$ to the graph of the given equation.

   a. $y = 4x^2 - 3$

   b. $y = -(x + 1)^2$

   b. $y = 9(x - 1)^2 + 3$

3. Transform the graph of $y = x^2$ into the graph of $y = ax^2 + bx + c$ where $a \neq 0$. Explain how this allows you to deduce the quadratic formula.

Related Common Core Standards:

G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

A-SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs.