Using Anchor Tasks to Drive Instructional Shifts as we Transition to the Common Core

Curtis Center Mathematics and Teaching Conference

March 1, 2014

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CHANGE
“To Improve is to Change”
-Winston Churchill
Is **CHANGE** the same as **TRANSITION**?

<table>
<thead>
<tr>
<th>CHANGE</th>
<th>TRANSITION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SITUATIONAL</strong></td>
<td><strong>PSYCHOLOGICAL</strong></td>
</tr>
<tr>
<td>Move to a new site</td>
<td>Adapting to the new environment</td>
</tr>
<tr>
<td>Retirement of a boss</td>
<td>Working with the new boss’ style</td>
</tr>
<tr>
<td>Reorganization of team member roles</td>
<td>Getting team members to work together productively</td>
</tr>
<tr>
<td>Adopting the Common Core State Standards</td>
<td>?</td>
</tr>
</tbody>
</table>
TRANSITION

The Neutral Zone

The New Beginning

Ending, Losing, Letting Go
“Because transition is a process by which people **unplug from an old world** and **plug into a new world**, we can say that **transition starts with an ending** and **finishes with a beginning**.”
Is **CHANGE** the same as **TRANSITION**?

**CHANGE** is fast, **TRANSITION** is slow.
“Changes of any sort...finally succeed or fail on the basis of whether the people affected do things differently. Do the employees let go of the old way of doing things, go through that difficult time between the old way and the new, and come out doing things the new way?”
Mathematics Sequence

Current Sequence
- Algebra 1
- Geometry
- Algebra 2
- College Prep
- GATE
- CST Exams

KHSD Common Core Bridge Materials
- Common Units with FOCUS on CCSS
- Big Ideas
- Mathematical Task Library
- Common KHSD Anchor Tasks
- Handouts, Teacher Notes, Video Lessons
- Enhanced Assessments
- Formative and Summative
- Professional Development to support all components

Common Core
- Course 1
- Course 2
- Course 3
- CCR
- College and Career Ready
- STEM
- SBAC Assessment
  - Spring 2015

FOCUS ◆ COHERENCE ◆ RIGOR ◆ RELEVANCE
Ramp Up the Rigor

Awareness

Instructional Practice
WHERE HAVE WE BEEN?
Awareness

What are the Common Core State Standards (CCSS)?

How are they similar to the California State Standards?

When will implementation begin?

How will the Common Core Standards be assessed?
WHERE HAVE WE BEEN?

Instructional Practice

Awareness

Ramp Up the Rigor
Ramp up the rigor of mathematical tasks for students.

Focus on instructional task analysis:

- PROTAGONIST
- CONCEPTS
- PROBLEM SOLVING
- COMMUNICATING
- REASONING
- DATA ANALYSIS
- MODELING
# Math Anchor Tasks

**KHSD Instructional Design Template for a Mathematical Task**

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<td>Students will organize the information with tables and graphs. They will then mathematically model the situation with diagrams, numerical quantities, variables, expressions, and/or equations.</td>
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<td>Students will formulate evidence-based predictions based on their multiple representations. They will evaluate their original guess.</td>
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<td>The teacher will use direct instruction to generalize the various student approaches. The mathematics will be formalized in a rigorous manner, as required by the content standards.</td>
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<td>The teacher and students can extend the discussion, pushing the mathematics above and beyond the learning targets of the task.</td>
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Students, when solving anchor tasks, apply the **Standards for Mathematical Practice**.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Anchor Task Template

Develop Mathematical Intuition

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Students will be presented with a context or situation where they will be asked to guess or predict: “What happens next?”
Anchor Task Template

2

Explore the Task

Students will explore the task’s context, make observations, and gather relevant information.
Model with Mathematics

Students will organize the information with tables and graphs. They will then mathematically model the situation with diagrams, numerical quantities, variables, expressions, and/or equations.
Apply the Mathematical Model

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Anchor Task Template

Communicate Mathematical Reasoning

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Students will precisely summarize their mathematical reasoning in words. They will share their own solution pathway and critique the reasoning of others.
The teacher will use direct instruction to generalize the various student approaches. The mathematics will be formalized in a rigorous manner, as required by the content standards.
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Math Anchor Tasks

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   Students will be presented with a context or situation where they will be asked to guess or predict: “What happens next?”

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   Students will explore the task’s context, make observations, and gather relevant information.

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   Students will organize the information with tables and graphs. They will then mathematically model the situation with diagrams, numerical quantities, variables, expressions, and/or equations.

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The Knowledge Taxonomy by Bloom was the first to define levels of cognition.
**Costa’s Levels of Questioning**

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**A Three Story Intellect!**

**BLOOM’S TAXONOMY** and Costa’s Levels of Questioning

The student will...

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<td>Learn specific facts, ideas, vocabulary, remembering/recalling information or specific facts.</td>
<td>Ability to grasp the meaning of material, communicate knowledge, understanding information without relating it to other material.</td>
<td>Ability to use learned material in new and concrete situations, use learned knowledge and interpret previous situations.</td>
<td>Ability to break down material into its component parts and perceive interrelationships.</td>
<td>Ability to put parts together to form a new whole; use elements in new patterns and relationships.</td>
<td>Ability to judge the value of material (statement, novel, poem, report, etc.) for a given purpose; judgment is based on given criteria.</td>
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**Introduction of knowledge**

**Level One—the basement**

By doing the following...

- collect, copy, define, describe, examine, find, group, identify, indicate, label, list, locate, match, name, omit, observe, point, provide, quote, read, recall, recite, recognize, repeat, reproduce, say, selected, sort, spell, state, tabulate, tell, touch, underline, who, when, where, what
- alter, associate, calculate, categorize, change, communicate, convert, distinguish, expand, explain, inform, name alternatives, outline, paraphrase, rearrange, reconstruct, relate, restate (own words), summarize, define, understand, verbalize, write

**Level Two—the ground floor**

By doing the following...

- acquire, adopt, apply, assemble, capitalize, construct, consume, demonstrate, develop, discuss, experiment, formulate, manipulate, organize, relate, report, search, show, solve novel problems, tell consequences, try, use, utilize
- analyze, arrange, break down, categorize, classify, compare, contrast, deduce, determine, diagram, differentiate, discuss causes, distinguish, draw, give reasons, order, separate, sequence, survey, take apart, test for, why

**Level Three—the penthouse**

By doing the following...

- alter, build, combine, compose, construct, create, develop, estimate, form a new, generate, hypothesize, imagine, improve, infer, invent, modify, plan, predict, produce, propose, reorganize, rewrite, revise, simplify, synthesize
- appraise, argue, assess, challenge, choose, conclude, criticize, critique, debate, decide, defend, discriminate, discuss, document, draw conclusions, editorialize, evaluate, grade, interpret, judge, justify, prioritize, rank, rate, recommend, reject, support, validate, weigh
Ramp Up the Rigor: Math

QUASAR:
Quantitative Understanding: Amplifying Student Achievement and Reasoning

Instructional Task:
a segment of classroom activity devoted to the development of a mathematical idea.
# Ramp Up the Rigor: Math

## Task Analysis Guide for Mathematics

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<td>- has no connection to the concepts or meanings that underlie the facts, rules, formulas, or definition being learned or reproduced.</td>
<td>- require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.</td>
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<th>Procedures Without Connections</th>
<th>Doing Mathematics</th>
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<td>- are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.</td>
<td>- requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).</td>
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<td>- require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.</td>
<td>- requires students to explore and understand the nature of mathematical concepts, processes, or relationships.</td>
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<td>- have no connection to the concepts or meaning that underlie the procedure being used.</td>
<td>- demands self-monitoring or self-regulation of one’s own cognitive process.</td>
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<td>- are focused on producing correct answers rather than developing mathematical understanding.</td>
<td>- requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.</td>
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<td>- require no explanations, or require explanations that focus solely on describing the procedure that was used.</td>
<td>- requires the students to analyze the task and actively examine task constraints that may limit possible solution strategies and solution.</td>
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<td>- requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solutions process required.</td>
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Ramp Up the Rigor: Math

Norman Webb, 1999

DOK Level

Depth of Knowledge

Assessments must be analyzed for both:

- Content
  - simple or complex

- Task Required
  - routine or non-routine

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**Mathematics Depth of Knowledge (DOK) Levels**

| Level 1 (Recall) | includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics, a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels, depending on what is to be described and explained. |

| Level 2 (Skill/Concept) | includes the engagement of some mental processing beyond an habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” “estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of objects or phenomena and then grouping or ordering the objects. Some action verbs, such as “explain,” “describe,” or “interpret,” could be classified at different levels depending on the object of the action. For example, interpreting information from a simple graph, or reading information from the graph, also are at Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited only to number skills, but may involve visualization skills and probability skills. Other Level 2 activities include noticing or describing non-trivial patterns, explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts. |

| Level 3 (Strategic Thinking) | requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and deciding which concepts to apply in order to solve a complex problem. |

| Level 4 (Extended Thinking) | requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas within the content area or among content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments and projects; developing and proving conjectures, making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs. |
Ramp Up the Rigor

Designing Math Tasks for Students

Dan Meyer

Set a low floor for entry, a high ceiling for exit. Write problems that require a simple first step but which stretch for miles.

Perplexity is the goal of engagement. What matters most is the question, “Is the student perplexed?” Our goal is to induce in the student a perplexed, curious state, a question in her head that math can help answer.

http://blog.mrmeyer.com
WHERE ARE WE NOW?

Ramp Up the Rigor

Instructional Practice

WHERE ARE WE NOW?
Instructional Practice

PLC Meetings
Department Meetings
Summer Projects
Site Administration Feedback
District Office Trainings
DOING MATH
CHANGE IN PEDAGOGY

A Balanced Mathematics Class

“WHERE” the mathematics works

Problem Solving and Modeling

DOING MATH

Conceptual Understanding

“WHY” the mathematics works

Procedural Skill and Fluency

“HOW” the mathematics works
# Math Anchor Tasks

## Three Stages

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td><strong>Heather Dallas</strong>&lt;br&gt;UCLA Curtis Center for Mathematics and Teaching</td>
<td><strong>Investigate</strong></td>
<td><strong>Conjecture</strong></td>
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<tr>
<td><strong>Dan Meyer</strong>&lt;br&gt;Stanford University&lt;br&gt;<a href="http://blog.mrmeyer.com">http://blog.mrmeyer.com</a></td>
<td><strong>Capture Perplexity</strong></td>
<td><strong>Share Perplexity</strong></td>
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<tr>
<td><strong>Phil Daro</strong>&lt;br&gt;Common Core State Standards Mathematics Writing Team</td>
<td><strong>You Do</strong>&lt;br&gt;A math task is posed to the class. The students are given time to try to solve the problem on their own in any way they can, using their own “panorama of prior knowledge.”</td>
<td><strong>We Do</strong>&lt;br&gt;Students are then allowed to tackle the task in teams. The different approaches of the groups are presented to the class and critiqued in a constructive mathematical discussion.</td>
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Mathematics Curriculum Committee

• **Summer, 2013**
  – 12 Teachers, chosen by application and interview
    • Recognized leaders
    • Algebra and Geometry experience
  – Creation of Quarterly Anchor Task problems for use district wide in Algebra and Geometry courses
    • Problems focus on problem solving with students demonstrating use of the 8 CCSS mathematical practices

<table>
<thead>
<tr>
<th>Kern High School District</th>
<th>Algebra 1 Anchor Tasks 2013-14</th>
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<tbody>
<tr>
<td><strong>1st Quarter:</strong></td>
<td><strong>The Border Problem</strong></td>
</tr>
<tr>
<td><strong>2nd Quarter:</strong></td>
<td><strong>Trike Wars</strong></td>
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<td><strong>3rd Quarter:</strong></td>
<td><strong>Pool Shark</strong></td>
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<td><strong>Flip It! Flip It Good!</strong></td>
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<td><strong>Last One Standing</strong></td>
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<td><strong>4th Quarter:</strong></td>
<td><strong>Lots of Plots</strong></td>
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- Finding and Expressing Patterns
- Linear Relationships
- 1. Exponential Growth Task
- 2. Exponential Decay Task A
- 3. Exponential Decay Task B
- Introduction to Quadratics
The Anchor Tasks

- Focus on the “Big Ideas” of each quarter
- Included for each task:
  - Detailed “Teacher Notes” to guide lesson
  - Ready-to-go PowerPoint for instruction
  - All necessary student handouts and worksheet

The Border Problem
Finding Patterns and Expressing Patterns

PowerPoint lesson slides:
The Border Problem

Student handouts:
The Border Problem - Act 1: PDF, Word
The Border Problem - Act 3: PDF, Word
The Border Problem - Encore: PDF, Word

The Border Problem - Formative Assessment: PDF, Word

Teacher Notes:
The Border Problem: PDF, Word

KHSD Instructional Design Template for Math Tasks
AnchorTasks.net
Math tasks that promote student engagement with the Standards for Mathematical Practice

Algebra Tasks

Geometry Tasks

The Standards for Mathematical Practice

Illustrative Mathematics
MARS Math Tasks
Inside Mathematics
Dan Meyer: Blog, Three Acts of a Mathematical Story, Tasks

California Common Core Math Standards
California Common Core Framework Chapters
Kern High School District Common Core Teacher Resources: Mathematics
Kern High School District
Pool Shark

Part 1

A Kern High School District Anchor Task
Predict how many games they played.

What number is too high?
What number is too low?
How many games did they play?
Are you making any assumptions about the games you didn’t watch when you were performing your calculations?
Marti said they played 256 games. Do you agree or disagree?

How do you think she arrived at that answer?

Justify your response.
Tony said they played 9 games. Do you agree or disagree?

How do you think he arrived at that answer? Justify your response.
Make a table that shows how Marti got her answer of 256

Make a table that shows how Tony got his answer of 9

Who’s correct?
Write a rule for Marti’s method and a rule for Tony’s method.
Are the rules correct?
Function
\( f(x) = 25 \cdot 2^x \)

Line
\( y = 25x \)
Function
\[ f(x) = 25 \cdot 2^{-x-1} \]

Line
\[ y = 25x \]

Point

A = (1, 25)
B = (2, 50)
Pool Shark

Part 2

A Kern High School District Anchor Task
Let’s say you wanted to make more money.

Would you rather increase the opening bet to $500 or change from “double or nothing” to “triple or nothing”? 
Without doing any math…yet
Who would rather increase the opening bet to $500?
Who would rather change doubling to tripling?

WHY??
Now, find out who’s right
What representation would be the best to support your answer?

(table, graph, equation, words)
Pass out Student Handout Part 2
For more information:

www.corestandards.org

CCSS English Language Arts Standards
and Literacy in History, Science, and Technical Subjects

CCSS Mathematics Standards

Literacy Standards for History, Science, and Technical Subjects
(Excerpt pages 99-166 of the CCSS ELA Standards)

Standards for Mathematical Practice
The varieties of expertise that mathematics educators should seek to develop in their students.

CCSS Mathematics: Appendix A
Designing High School Mathematics Courses based on the CCSS

CCSS ELA: Appendix A
Research Supporting Key Elements of the Standards and Glossary of Terms

CCSS ELA: Appendix B
Text Exemplars and Sample Performance Tasks

CCSS ELA: Appendix C
Samples of Student Writing

California Department of Education
Common Core Resources

California CCSS ELA Standards
and Literacy in History, Science, and Technical Subjects

California CCSS Mathematics Standards

California CCSS Implementation Timeline
California CCSS Implementation Plan (March 2012, Word document, 42 pages)

2013 Revision of the California Mathematics Framework

2014 Revision of the California English Language Arts
English Language Development Framework

California CCSS Frequently Asked Questions
California SBAC Assessment Resources
California SBAC Frequently Asked Questions

www.kernhigh.org/instruction/commoncore
Using Anchor Tasks to Drive Instructional Shifts as we Transition to the Common Core

Curtis Center Mathematics and Teaching Conference

March 1, 2014

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# Ramp Up the Rigor: Math

## TASK ANALYSIS GUIDE FOR MATHEMATICS

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<tr>
<td>- are algorithmic. Use of the procedure is either specifically called for or its used is evident based on prior instruction, experience, or placement of the task.</td>
<td>- requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).</td>
</tr>
<tr>
<td>- require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.</td>
<td>- requires students to explore and understand the nature of mathematical concepts, processes, or relationships.</td>
</tr>
<tr>
<td>- have no connection to the concepts or meaning that underlie the procedure being used.</td>
<td>- demands self-monitoring or self-regulation of one’s own cognitive process.</td>
</tr>
<tr>
<td>- are focused on producing correct answers rather than developing mathematical understanding.</td>
<td>- requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.</td>
</tr>
<tr>
<td>- require no explanations, or require explanations that focus solely on describing the procedure that was used.</td>
<td>- requires the students to analyze the task and actively examine task constraints that may limit possible solution strategies and solution.</td>
</tr>
<tr>
<td></td>
<td>- requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solutions process required.</td>
</tr>
</tbody>
</table>
### Level 1 (Recall)
Includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics, a one-step, well defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels, depending on what is to be described and explained.

### Level 2 (Skill/Concept)
Includes the engagement of some mental processing beyond an habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” “estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of objects or phenomena and then grouping or ordering the objects. Some action verbs, such as “explain,” “describe,” or “interpret,” could be classified at different levels depending on the object of the action. For example, interpreting information from a simple graph, or reading information from the graph, also are at Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited only to number skills, but may involve visualization skills and probability skills. Other Level 2 activities include noticing or describing non-trivial patterns, explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

### Level 3 (Strategic Thinking)
Requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and deciding which concepts to apply in order to solve a complex problem.

### Level 4 (Extended Thinking)
Requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas within the content area or among content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments and projects; developing and proving conjectures, making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.
CHANGE IN PEDAGOGY

A Balanced Mathematics Class

“WHERE” the mathematics works

Problem Solving and Modeling

“HOW” the mathematics works

Procedural Skill and Fluency

“WHY” the mathematics works

Conceptual Understanding

DOING MATH
AnchorTasks.net

Math tasks that promote student engagement with the Standards for Mathematical Practice

Algebra Tasks

Geometry Tasks

The Standards for Mathematical Practice

Illustrative Mathematics
MARS Math Tasks
Inside Mathematics
Dan Meyer: Blog, Three Acts of a Mathematical Story, Tasks

California Common Core Math Standards
California Common Core Framework Chapters
Kern High School District Common Core Teacher Resources: Mathematics
Kern High School District
At this point you have already worked on the problem and should have a good idea of the answer. In the space below use the Rule of 4 to re-write the correct answer using a Table, a Graph, an Equation and in Words. Be sure to label all variables, label your graph and use complete sentences. Express your answer in context of the problem.
Rule of Four
Multiple Representations of Mathematical Functions

1. Verbal representations: **WORDS**
2. Numerical representations: **TABLES**
3. Graphical representations: **GRAPHS**
4. Symbolic representations: **EQUATIONS**