EXPLORING ADDITION FACT STRATEGIES

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Debbie Leslie and Denise Porter

CENTER FOR ELEMENTARY MATHEMATICS AND SCIENCE EDUCATION
UNIVERSITY OF CHICAGO
Goals

• Develop a definition for basic fact fluency

• Become familiar with the fact strategies used in *Everyday Mathematics* and the research behind those strategies.

• Look at an *Everyday Mathematics* lesson
• What do you recall from your fact development experience as a child?

• What types of activities/tools do you recall using to build fact automaticity?
What do you think it means to have fluency with basic facts?
WHAT IS FACT FLUENCY?

POLL: WHAT IS FACT FLUENCY?

Which of these describes how you think of fact fluency? Click
From the Common Core State Standards, Grade 1:
1.0A6:
Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

From the Common Core State Standards, Grade 2:
2.0A2:
Fluently add and subtract within 20 using mental strategies (reference to Grade 1 expectation). By end of Grade 2, know from memory all sums of two one-digit numbers.
From the Common Core State Standards, Grade 1:  
1.0A6:  
Add and subtract within 20, *demonstrating fluency* for addition and subtraction within 10. *Use strategies* such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

From the Common Core State Standards, Grade 2:  
2.0A2:  
*Fluently* add and subtract within 20 using mental *strategies* (reference to Grade 1 expectation). By end of Grade 2, *know from memory* all sums of two one-digit numbers.
This would suggest that *fluency is different* from automatic retrieval. Research heavily supports this…

So what does fluency *really* mean?
Fluency with basic facts is the efficient, appropriate, and flexible application of single-digit calculation skills and is an essential aspect of mathematical proficiency (Baroody, 2006).

Furthermore…

Fluency with basic number combinations develops from well-understood meanings of the four operations and from a focus on thinking strategies (NCTM Principles and Standards for School Mathematics, 2000).
A fluency approach to learning basic facts recognizes the phases students must progress through as they develop mastery with facts (from Baroody, 2006):

• Phase 1: Modeling and/or counting to find the answer
• Phase 2: Deriving answers using reasoning strategies based on known facts
• Phase 3: Mastery: efficient production of answers
Research Summary

• http://everydaymath.uchicago.edu/about/research-results/
How do we achieve fluency in first and second grade?

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With a focus on strategies, *not* rote memorization

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“Duplicates” if students understand the commutative property

Near Doubles (Doubles +/- 1 or 2)

Relates to meaning of addition or counting sequence

Combinations of 10

Doubles

Pairs that are easy to make 10 with
Ten Frames
Lesson 4-8

Combinations of 10

Overview  Children record addition facts they know and use combinations of 10 as a strategy for adding and subtracting within 20.
# Combinations of 10

**Overview:** Children record addition facts they know as a strategy for adding and subtracting within 20.

**Before You Begin**
- For Part 1, sequence Quick Look Cards 60, 64, and 68.
- For Part 2, sequence Quick Look Cards 88, 92, 94, 96, and 98.

**Vocabulary**
- addition fact
- combination of 10

## Materials
- Quick Look Cards

## Warm Up

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<tr>
<th>Time</th>
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<tr>
<td>15-20 min</td>
<td>Mental Math and Fluency</td>
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<tr>
<td>20-30 min</td>
<td>Daily Routines</td>
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<tr>
<td>20-30 min</td>
<td>Math Message</td>
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<tr>
<td>20-30 min</td>
<td>Fast Strategy Review</td>
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## Focus

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<tr>
<td>10-15 min</td>
<td>Math Message</td>
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<td>30-40 min</td>
<td>Math Journal 1, p. 57</td>
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<tr>
<td>20-30 min</td>
<td>Introduction to Combinations of 10</td>
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## Spiral Snapshot

**GMC:** Add combinations of 10 automatically.

## Practice

<table>
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<tr>
<th>Time</th>
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<tr>
<td>15-20 min</td>
<td>Playing High Roller</td>
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<tr>
<td>15-20 min</td>
<td>Math Ovals 4-6: Review for Unit 5</td>
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<td>15-20 min</td>
<td>Home Link 4-6</td>
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<tr>
<td>15-20 min</td>
<td>Homework</td>
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**Common Core**
- 1.OA.6
- 1.OA.3, 1.OA.6
- SMP1, SMP6
- SMP2, SMP7

See pages xiv–xvii.
Differentiation Options

Readiness 5-10 min
Two-Flipped Penny Addition
Math Masters, p. 29 (optional), 10 pennies, slate
To review finding pairs of numbers that add to 10, have children do Two-Flipped Penny Addition. Starting with a pile of 10 pennies, children group come in one hand and the rest in the other. They report to their partners, “I have ___ in one hand and ___ in my other hand. I have 10 pennies in all.” Children record the combinations on their slates.

Enrichment 5-10 min
Fact Wizard Activity Grid 59, timer or stopwatch
To provide practice with addition and subtraction facts, have partners record as many facts as they can in one minute. Partners check the accuracy of each other’s facts.

Extra Practice 10-15 min
Ice Cubes in the Cup
Math Masters, p. 171, per partner (a ten-frame, plastic cup, and 10 counters)
To provide practice finding missing addends, have children use the ten-frame and counters to figure out the number of “Ice cubes” (counters) in the cup. By using 10 ice cubes, children specifically find missing numbers that add to 10, which provides valuable fluency practice with combinations of 10.

1. The first partner secretly hides between 0 and 10 ice cubes in the cup.
2. The second partner then places the remaining “Ice cubes” in the “Ice tray” (ten-frame) and looks at the empty spaces to determine the number of ice cubes in the cup.
3. Using the record sheet, partners record the number of ice cubes in the cup, the number of ice cubes in the tray, and a number sentence for that round.
4. Partners trade roles and repeat.

English Language Learners Support
Beginning ELL: Think aloud and demonstrate combining two sets of objects.
Intermediate ELL: Think aloud and demonstrate combining two sets of objects with other combinations of shapes. Because children to repeat your statement, direct children to make combinations of specific shapes and say, “I have a combination of ___ and ___.”
Standards and Goals for Mathematical Practice

SMP1 Make sense of problems and persevere in solving them.
GMP1.6 Compare the strategies you and others use.

SMP2 Reason abstractly and quantitatively.
GMP2.2 Make sense of the representations you and others use.

SMP7 Look for and make use of structure.
GMP7.1 Look for mathematical structures such
Warm-Up Part 1

Mental Math and Fluency
Do Quick Looks with children. Display Quick Look Cards in this order: 60, 64, 69. Show children each image for 2–3 seconds and then remove or cover it. Have children share what they saw and how they saw it.

GMP2.2, GMP6.1 Ask questions, such as: What is 1 more (or less) than you saw? For detailed instructions, see Lesson 1-1.

Daily Routines
Have children complete the Daily Routines. For detailed instructions, see pages 2–37. For specifics on CCSS coverage, see pages xiv–xvii.
Mental Math and Fluency
Focus  Part 2

Math Message
Solve $5 + 0 = \underline{}$. Record how you solved it and compare your strategy with a partner.

Fact Strategy Review
*Math Journal 1, p. 95*

**Math Message Follow-Up** Ask children to explain how they found the sum of $5 + 0$. Sample answer: When I add 0, I am adding nothing, so the number stays the same. The answer is 5. Though children do not need to use the term, help them internalize 0 as the Additive Identity, meaning that the sum of 0 and a number is that number ($a + 0 = a$). Make sure children clearly articulate what adding zero means and how that helps them solve $+0$ number sentences. **GMP7.1**

Explain that **addition facts** are two numbers from 0 to 10 and their sum. Point out that children can use the strategies on the Strategy Wall to solve addition facts. Record children’s ideas about adding 0 on the Strategy Wall.
Math Message

Solve $5+0 =$

Record how you solved it and compare your strategy with a partner.
Fact Strategy Wall

- **+0**
  
  Answer is the number added to zero.
  
  \[5 + 0 = 5\]

- **+1**
  
  Answer is the number that comes after.
  
  \[3 + 1 = 4\]

- **Counting on from the larger number**
  
  Start with the larger number. Count on the smaller number.
  
  \[3 + 2 \rightarrow 3 \quad 4 \quad 5\]
  
  \[3 + 2 = 5\]
### My Facts Inventory Record, Part 1 (continued)

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<th>Don't Know It</th>
<th>How I Can Figure It Out . . .</th>
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1.OA.6
Introducing Combinations of 10
Assessment Check-In

Listen to how children describe finding the total number of dots in Quick Looks. Expect them to start mentally combining dots in the ten frames rather than counting each dot as they begin to see dots in combinations of 10. **GMP2.2** Children who still count every dot may benefit from doing Quick Looks in small groups to allow them to develop and share their thinking.

Assessment and Reporting Go Online to record student progress and to see trajectories toward mastery for these standards.
Practice Part 3

► **Playing High Roller**

*Math Masters, p. 527*

Have children play *High Roller* to practice adding numbers. For more detailed instructions, see Lesson 2-6.

**Observe**

- Which children count on to find the sum of the dice?

**Discuss**

- Was it easier to count on with certain sums? Which ones?

► **Math Boxes 4-8: Preview for Unit 5**

*Math Journal 1, p. 59*

**Mixed Practice** Math Boxes 4-8 are paired with Math Boxes 4-12. These problems focus on skills and understandings that are prerequisite for Unit 5. You may want to use information from these Math Boxes to plan instruction and grouping in Unit 5.

► **Home Link 4-8**

*Math Masters, pp. 112-113*

**Homework** Children take home a Family Letter about addition and subtraction facts. They practice finding combinations of 10 by doing a version of Two-Fisted Penny Addition with someone at home.
High Roller

Directions

1. Take turns. Roll both dice.
2. Record your first roll in the first two squares.
3. Cross out the smaller number.
4. Roll the die that shows the smaller number again.
5. Record your second roll in the third square.
6. Record the sum of the two dice on the line.
7. The player with the larger sum wins the round.
8. Play 5 rounds. The player who wins the most rounds is the winner.

Materials

2 dice
High Roller Record Sheet
(Math Masters, page G27)

Players

2

Skill

Adding numbers

Object of the Game

To have the larger sum
### Record Sheet

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Fact Triangles

Addition/Subtraction Fact Triangle

17
8, 9

17
8, 9

Addition/Subtraction Fact Triangle
A Wide Variety of Practice to Promote Fact Power
Games Provide Motivating Practice
Fact Power

• “Automatically knowing basic facts is as important to learning mathematics as knowing words by sight is to reading.”
  – EM authors

• Developing computational fluency is strongly emphasized in EM.
THANK YOU

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Debbie Leslie daleslie@uchicago.edu
Denise Porter porterd@uchicago.edu

CENTER FOR ELEMENTARY MATHEMATICS AND SCIENCE EDUCATION
UNIVERSITY OF CHICAGO