Area Formulas…How did they Figure them out?

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(Common Core standards 6.G.1).
Students themselves develop formulas for computing the area of triangles, trapezoids, kites, parallelograms and circles by composing into rectangles or decomposing into triangles and other shapes.

MP7
notice and use structure

MP3
construct viable arguments

Background:
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A husband-and-wife team of Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof, noticed the difficulties that their students had in learning geometry. These observations led them to develop a theory involving levels of thinking in geometry that students pass through as they progress from merely recognizing a figure to being able to write a formal geometric proof. Their theory explains why many students encounter difficulties in their geometry course, especially with formal proofs. The van Hieles believed that writing proofs requires thinking at a comparatively high level, and that many students need to have more experiences in thinking at lower levels before learning formal geometric concepts.

Frequently Asked Questions
Q. What are the van Hiele levels of geometry understanding?
A. There are five levels, which are sequential and hierarchical. They are:
Level 1 (Visualization): Students recognize figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.
Level 2 (Analysis): Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.
Level 3 (Abstraction): Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.
Level 4 (Deduction): Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.
Level 5 (Rigor): Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. Students at this level can understand the use of indirect proof and proof by contrapositive, and can understand non-Euclidean systems. Clements and Battista (1992) also proposed the existence of Level 0, which they call pre-recognition. Students at this level notice only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures. For example, they may distinguish between triangles and quadrilaterals, but may not be able to distinguish between a rhombus and a parallelogram.
Area of Polygonal Regions

Basic Postulates:

I. The area of a rectangle is equal to the product of the ______ and the ______ for that base.
   The formula is ______________

II. Every __________ ___________ has an area.

III. If two closed figures are _________________, then their areas are ___________.

   If \[ \text{ } \] \[ \approx \] \[ \Rightarrow \] \[ A_1 = A_2 \]

IV. If two closed regions intersect only along a common boundary (that is, the regions don't overlap), then the area of their ____________ is equal to the ______ of their individual _________.

   \[ A_3 = A_1 + A_2 \]
A. Area of Parallelograms

\[ \text{Area} = b \times h \]

B. Area of Triangles

\[ \text{Area} = \frac{1}{2} b \times h \]

C. Area of Trapezoids

\[ \text{Area} = \frac{1}{2} (b_1 + b_2) \times h \]
D. Area of Kites

\[ \text{Area of Kite} = \frac{1}{2} \times d_1 \times d_2 \]
Area of Regular Polygons

I. Regular polygons

In a regular polygon all interior ___________ are congruent and all ___________ are congruent.

In regular polygon PENTA at right, O is the center

ΟΑ is a radius

ΟΜ is an apothem

So . . . A(n) __________ of a regular polygon is a segment joining the center to the midpoint of any side of the polygon.

and A(n) __________ of a regular polygon is a segment joining the center to the vertex of any angle of the polygon.

<table>
<thead>
<tr>
<th>Properties of Radii and Apothems</th>
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<tbody>
<tr>
<td><strong>All apothems of a regular polygon are ______________.</strong></td>
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<tr>
<td><strong>Only__________ polygons have an apothem.</strong></td>
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<tr>
<td><strong>An apothem is a radius of a circle ________________ in the polygon.</strong></td>
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<tr>
<td><strong>An apothem is the ________________ ________________ of a side.</strong></td>
</tr>
<tr>
<td><strong>A radius of a regular polygon is a radius of a circle ________________ about the polygon.</strong></td>
</tr>
<tr>
<td><strong>A radius of a regular polygon ________________ an angle of the polygon.</strong></td>
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</tbody>
</table>
Each figure below is a regular polygon. Using dotted lines, draw all the radii of the polygon. How many triangles are formed in each polygon and what is true about the triangles formed?

______________________________________________________

Draw in an altitude of one triangle in each polygon and label each of them with an h.

Now write an expression for the area of each of the regular polygon in terms of the areas of the triangles that you have formed.

\[ A = \frac{1}{2} \times \text{base} \times \text{height} \]

What is another name for the first constant times the base? __________

If we allow \( p \) to represent the answer to the previous question and we use \( a \) as the apothem of each figure (instead of \( h \)), write a general formula for the area of any regular polygon.

\[ A = \frac{1}{2} \times \text{apothem} \times \text{perimeter} \]

Write this formula in words: ________________________________

______________________________________________________

______________________________________________________
The limit of the perimeter of regular polygons as the number of sides increases approaches the circumference of the circumscribed circle.

\[ A_{\text{polygon}} = \frac{1}{2} p a \]
\[ A_{\text{circle}} = \frac{1}{2} \]

The limit of the perimeter of regular polygons as the number of sides increases approaches the circumference of the circumscribed circle.
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