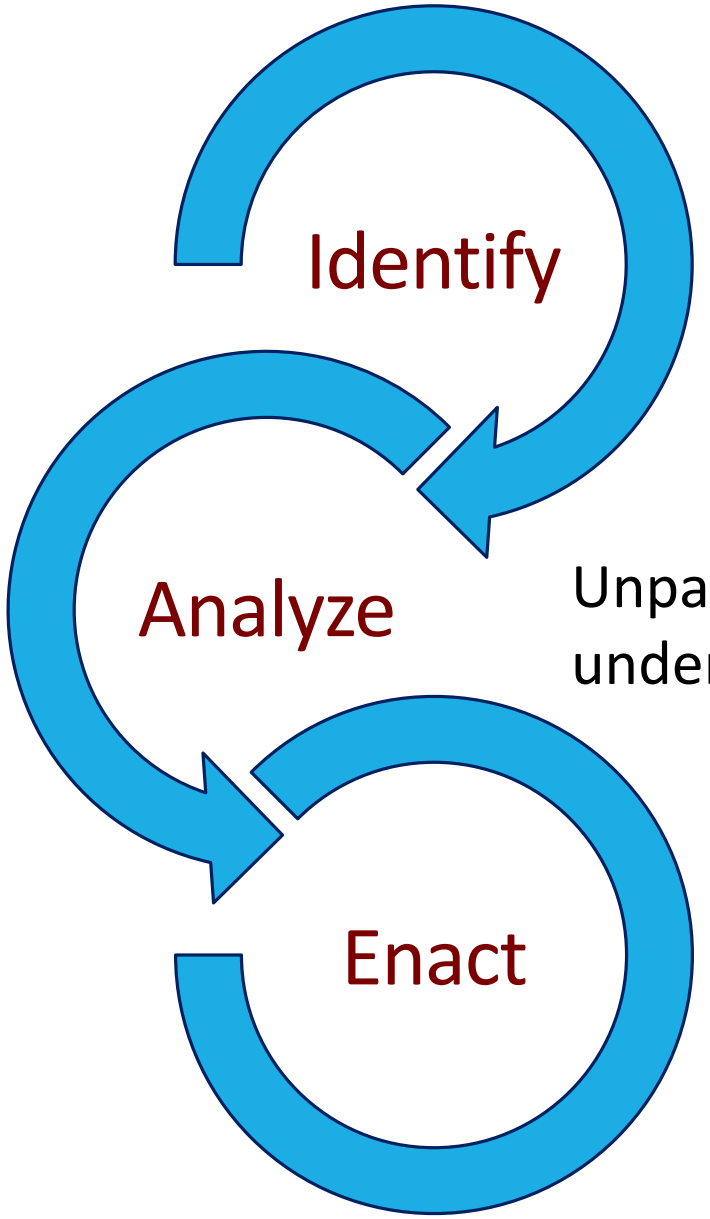
MDTP
A Tool For Teachers

Identify, Analyze, and Enact: Using MDTP Formatively

CSU/UC Mathematics Diagnostic Testing Project

DR. KIMBERLY SAMANIEGO, DIRECTOR



Identify students' current mathematical understandings, misconceptions, and gaps in content knowledge

Unpack the progressions of mathematics needed to build the essential understandings that students need to access and master the content

Enact strategies and design learning experiences to support learning goals

Question 17 Assessment of Preparedness for 7th Grade Mathematics (7M40A15)

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What is the distance between the points located at 7 and $-1\frac{1}{2}$ on the number line?

Determine the solution and discuss ways that students might answer this problem.

Question 17 Assessment of Preparedness for 7th Grade Mathematics (7M40A15)

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What is the distance between the points located at 7 and $-1\frac{1}{2}$ on the number line?

		Student Percentage
A	$5\frac{1}{2}$	22%
B	$6\frac{1}{2}$	44%
C	$7\frac{1}{2}$	6%
D	$8\frac{1}{2}$	28%

Omitted) 0% Not Seen) 0%

- **Is this item a strength for some students? Explain.**
- **Is this item an overall strength for the class? Why or why not?**

Classifying Items

Analysis Guide		
Determine Strengths (Item Analysis)	Determine Common Misconceptions and Errors (Distractor Analysis)	Determine Areas of Unfinished Learning and/or Gaps of Content (Item Analysis)
1. Items where most of the class chose the correct solution. 2. Analyze the concept and/or skill of each item. 3. Determine the relevance to the content of this course and how this knowledge informs planning your (curricular/unit/lesson). 4. Ask: Now what?	1. Items where a large percent of the class chose an incorrect option. 2. Analyze the error found in this option (distractor). 3. Determine the relevance to the content of this course and how this knowledge informs your planning (curricular/unit/lesson). 4. Ask: Now what?	1. Items where all options are chosen somewhat evenly or that students omitted at a high rate. 2. Analyze the concept and/or skill of each item. 3. Determine the relevance to the content of this course and how this knowledge informs planning your (curricular/unit/lesson). 4. Ask: Now what?

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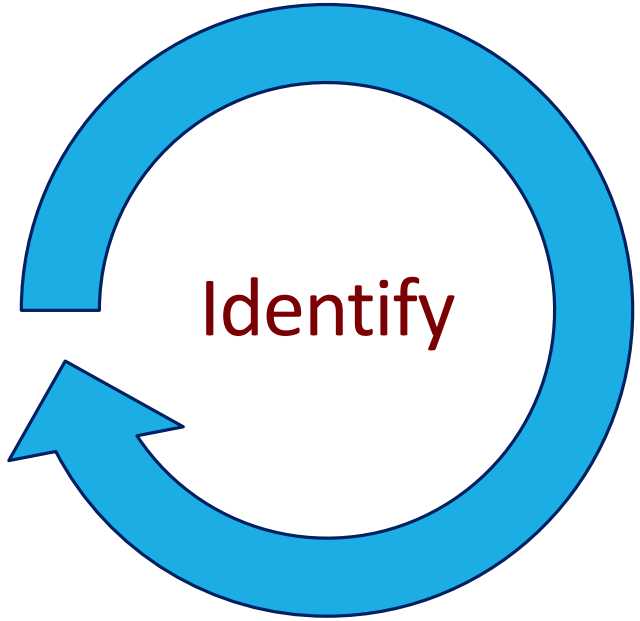

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Identify students' current mathematical understandings, misconceptions, and gaps in content knowledge

- What do students know? How can I build on this understanding?
- Describe any (mis)conceptions.
- Describe any potential gaps of content knowledge. Where will students see this in my current curriculum?

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Unpack the progressions of mathematics needed to build the essential understandings that students need to access and master the content

- Where do students first see this?
- What is the mathematics that students need to know to understand the concept of that math?
- Where do students need to go?
- What are the *implications* of doing nothing?

Question 37 High School Readiness Test (HS45A15)

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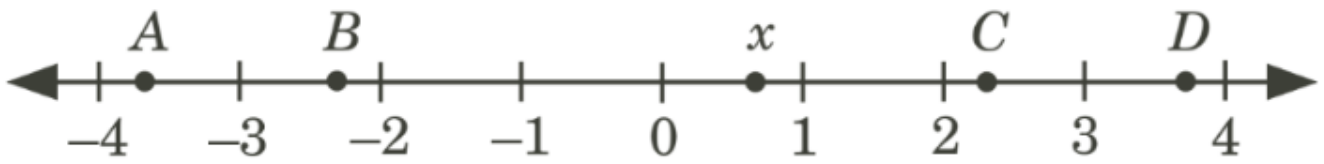
A number x is between 0 and 1. Which of the following points on the number line shown below could be the location of $-x + 3$?



Student Percentage

A	A	9%
B	B	37%
C	C	31%
D	D	6%

Omitted) 14% Not Seen) 3%

Share out:

- What do students know (what can you build on)?
- What is a (mis) conception and what is the math that you need to unpack so that students can be successful in the content in your course?
- What is a potential gap, and when do you address this content in your current instruction?

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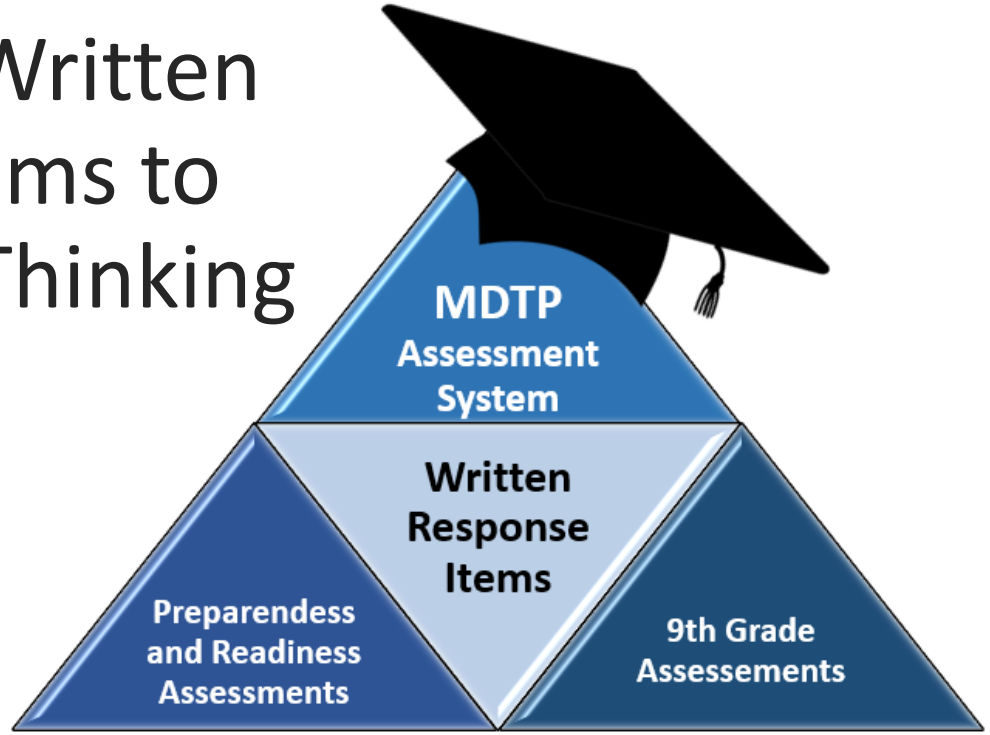
Using MDTP Written Response Items to Elicit Student Thinking



MDTP Written Response

- Solve the AR96LINE. Work individually for 3-5 minutes.
  - Share out with your table group. ***Listen for different ways of thinking.***
  - Choose one way of thinking and record this method to share with other table groups.
  - Explain this method to the group.

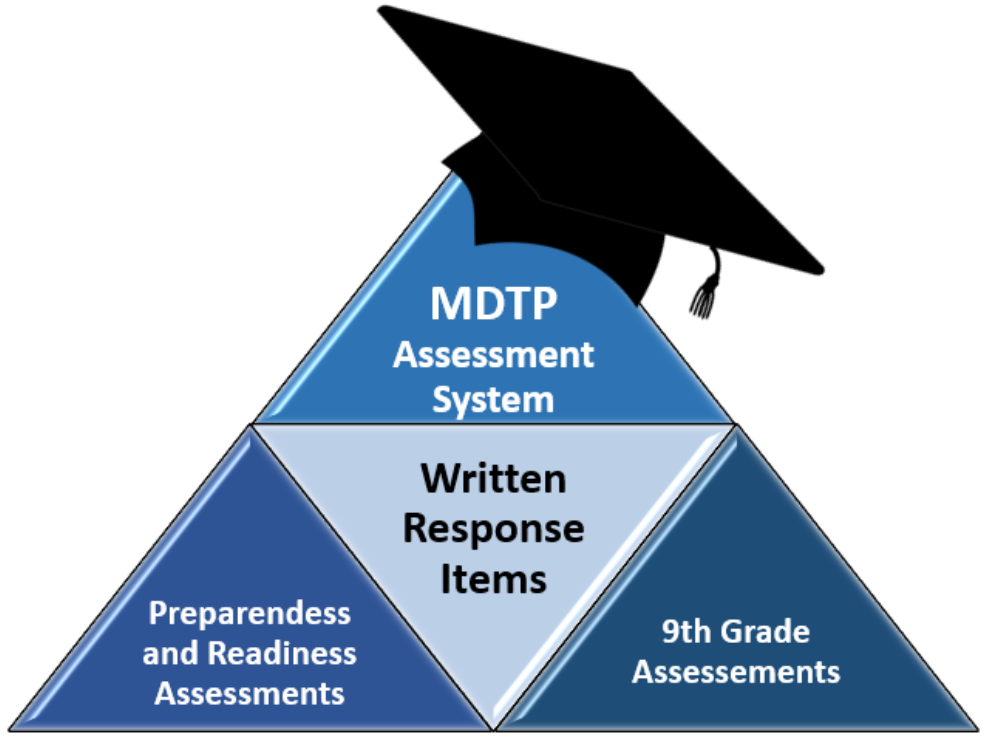
Student Name _____ Class _____ Score

For this written response activity, your work should clearly show how you solved each part. Label any figures you draw.

A. On the positive portion of the number line shown above, find a point and label it P so that the fraction $\frac{5}{P}$ is less than 1. (Be sure to mark the point P and not the fraction $\frac{5}{P}$.) Are there any other locations for P? If so, describe the location of all these points. If not, why not?

B. On the positive portion of the number line shown above, find a point and label it Q so that the fraction $\frac{5}{Q}$ is greater than 1. Are there any other locations for Q? If so, describe the location of all these points. If not, why not?

C. On the positive portion of the number line shown above, find a point and label it R so that the fraction $\frac{5}{R}$ is equal to 1. Are there any other locations for R? If so, describe the location of all these points. If not, why not?



Number Line Essence Statement

Algebra Readiness: Graphical Representation

The task is to mark and label on a given number line points that meet three given specifications and support these choices with explanations. All require conceptual understanding of the number line, rational numbers, and inequalities. To fully accomplish the task, student work must be correct and clearly presented.

- To find a location for a point P so that $\frac{5}{P}$ is less than 1, student work will show that any such point must be positioned to the right of 5. Student work will demonstrate an understanding that any number greater than 5 will make the value of $\frac{5}{P}$ less than 1.

- To find a location for a point Q so that $\frac{5}{Q}$ is greater than 1, student work will show any such point must be positioned on the number line between 0 and 5. Student work will demonstrate an understanding that any number between 0 and 5 will make the value of $\frac{5}{Q}$ greater than 1.

- To find a location for the point R so that $\frac{5}{R} = 1$, student work will show the label 'R' at the point 5 on the number line. Student work will demonstrate recognition of the uniqueness of this location within the response to the question.

POSSIBLE EXTENSIONS FOR CLASS ACTIVITY:

Discuss the fact that the solution set does not just include integers.

Discuss the implication upon the solution if the number line is extended to include negative numbers.

General Scoring Rubric

Score	Category	Description
0	No Response	Not attempted or incorrect or off task
1	Minimal	Incomplete response with major errors
2	Partial	Reasonable approach but not well developed
3	Satisfactory	Generally well developed and presented, but contains some omissions or errors
4	Excellent	Complete, correct, and clearly stated

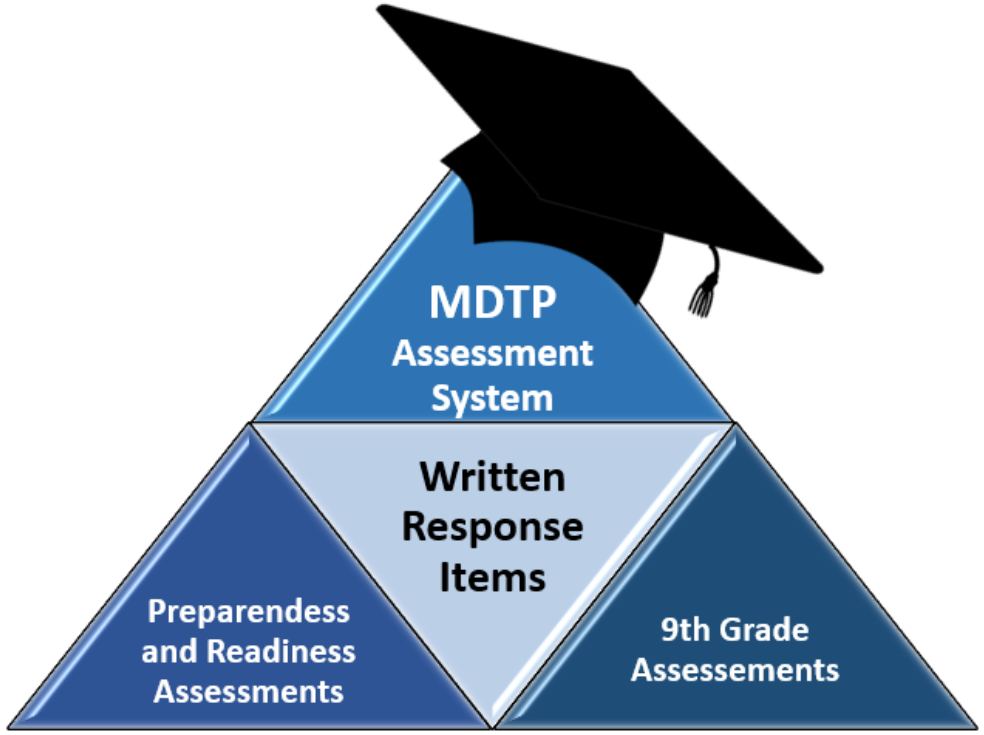
RUBRIC

Notes:

- For a score of 2 or 3, explanations and descriptions limited to the natural numbers will be accepted as complete.
- For a score of 4, explanations and descriptions may not be limited to natural numbers.
- Simply stating "Only one possible location for R" with no further explanation does not constitute a complete answer to Part C.
- In this rubric "part" always refers to Part A or Part B or Part C of the problem.

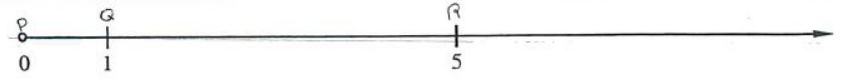
Score	Description
1	Correct locations for at least one point for one, two, or three parts.
2	Correct answer to one part OR correct locations for at least one point for each part and partial description of locations of the other points for one part.
3	Complete answer to one part and correct locations for at least one point for each of the other parts OR complete answers to two parts.
4	Complete answers to all three parts.

Note: See General Scoring Rubric for Written Response Items for further guidelines.



Student Name _____ Class Algebra 1 Per. 2 | 3 Score

For a complete response: **express** your thinking in words; **label** any figures you draw; **identify** any formulas you use; **make clear** the source of any numbers you use.



A. On the positive portion of the number line shown above, find a point and label it P so that the fraction $\frac{5}{P}$ is less than 1. (Be sure to mark the point P and not the fraction $\frac{5}{P}$.) Are there any other locations for P? If so, describe the location of all these points. If not, why not? No, there are no other locations for the P because if P = 0 then $\frac{5}{P}$ would equal to 0 which would be less then 1. There is no other point because then it would be greater then 1.

B. On the positive portion of the number line shown above, find a point and label it Q so that the fraction $\frac{5}{Q}$ is greater than 1. Are there any other locations for Q? If so, describe the location of all these points. If not, why not? Yes, there are other locations for the Q, and the other locations are $\frac{5}{2}, \frac{5}{3}, \frac{5}{4}$. These locations are greater then 1.

C. On the positive portion of the number line shown above, find a point and label it R so that the fraction $\frac{5}{R}$ is equal to 1. Are there any other locations for R? If so, describe the location of all these points. If not, why not? No, there are no other location for the R because $\frac{5}{6}$ would be less then 1 and $\frac{5}{4}$ would be greater then one. so the only location for R is 5.

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4	Complete answers to all three parts.

Note: See General Scoring Rubric for Written Response Items for further guidelines.

Ideas for Supporting Students' Use and Understanding of Rubric Scoring

- Display the General Rubric before students solve the problem and explain what students' responses need to show for a score of 4.
- Teacher-scoring (students revise)
  - Teachers score student work, return the specific rubric with student work, read each of the rubric components to students, and then allow students time to revise responses as needed.
- Self-scoring (students score and revise)
  - Allow students to score their own work using the specific rubric and give them time to revise responses as needed.

Types of Engagement

- **Introducing**
  - When students are first learning the content and are exploring their strategies and developing their skills sets
- **Advancing**
  - When students have spent time in the content and are advancing their knowledge and are incorporating diverse and flexible skill sets
- **Re-engaging**
  - When activating students' prior knowledge to launch a new concept/skill or
  - When a common misconception has been identified and ***needs*** intervention

Re-Teaching vs. Re-Engagement

Re-teaching

Teaching the unit again

Addressing the same missed basic skills

Do the same problems over more practice

Learn procedures focused mostly on underachievers

Cognitive load usually lower

VS.

Re-engaging

Revisiting student thinking

Addressing conceptual understanding

Examine the task from different perspectives

Critique approaches

Make connections

Engage entire class in mathematics

Cognitive load usually higher

Inside Mathematics: http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons

Strategies for Engagement

Introducing: When students are first learning the content and are exploring their strategies and developing their skills sets

- Students solve a problem independently. Monitor students' progress and choose a few students to present their ways of thinking
  - Purposely align student presentations to lead into the upcoming lesson
  - Anticipate errors and deliver content in response to errors
  - Assess learning of new content and provide feedback using rubric

Strategies for Engagement

Advancing: When students have spent time in the content and are advancing their knowledge and are incorporating diverse and flexible skill sets

- Students work collaboratively to solve the problem. Through discussion, justification, argument, and debate students agree on one solution and present/submit this one solution
  - Poster Walks
  - Chart and name novel methods and class-validated methods
  - Assess student understanding, build on charted methods, and remedy errors or misconceptions (see re-engagement)

Strategies for Engagement

Re-engaging: When activating students' prior knowledge to launch a new concept/skill *OR* when a common misconception has been identified and *needs* intervention

- Activate Prior Knowledge:
  - Take pulse and design lessons to meet students where they are (anticipate and prepare for potential errors)
- Remedy Misconceptions
  - Complete only one section and return every few days to distribute the learning over time
  - Chart student responses and keep charts visible for continued reference
  - Make connections to the current learning

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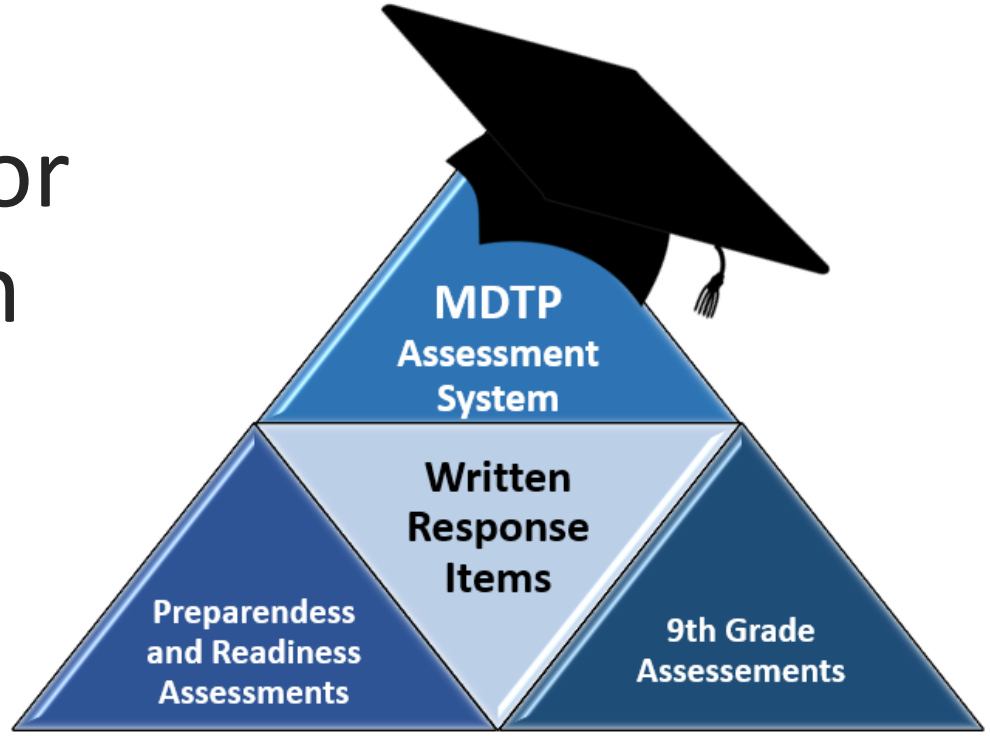
Learning for Retention



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You Have Data; Now What?

According to NCTM, it is important "to identify and ***address potential gaps*** and ***misconceptions*** when it matters most to students, which is during instruction, before errors or faulty reasoning becomes consolidated and more difficult to remediate" (National Council of Teachers of Mathematics, 2014, p.53).

However, "if a learner practices a skill incorrectly, but well, unlearning and relearning that skill correctly is very difficult" and the degree of success is dependent on the age of the learner, the period of practice, and the learner's motivation to change" (Sousa, 2017, p.112).

Learning and Practice

Massed

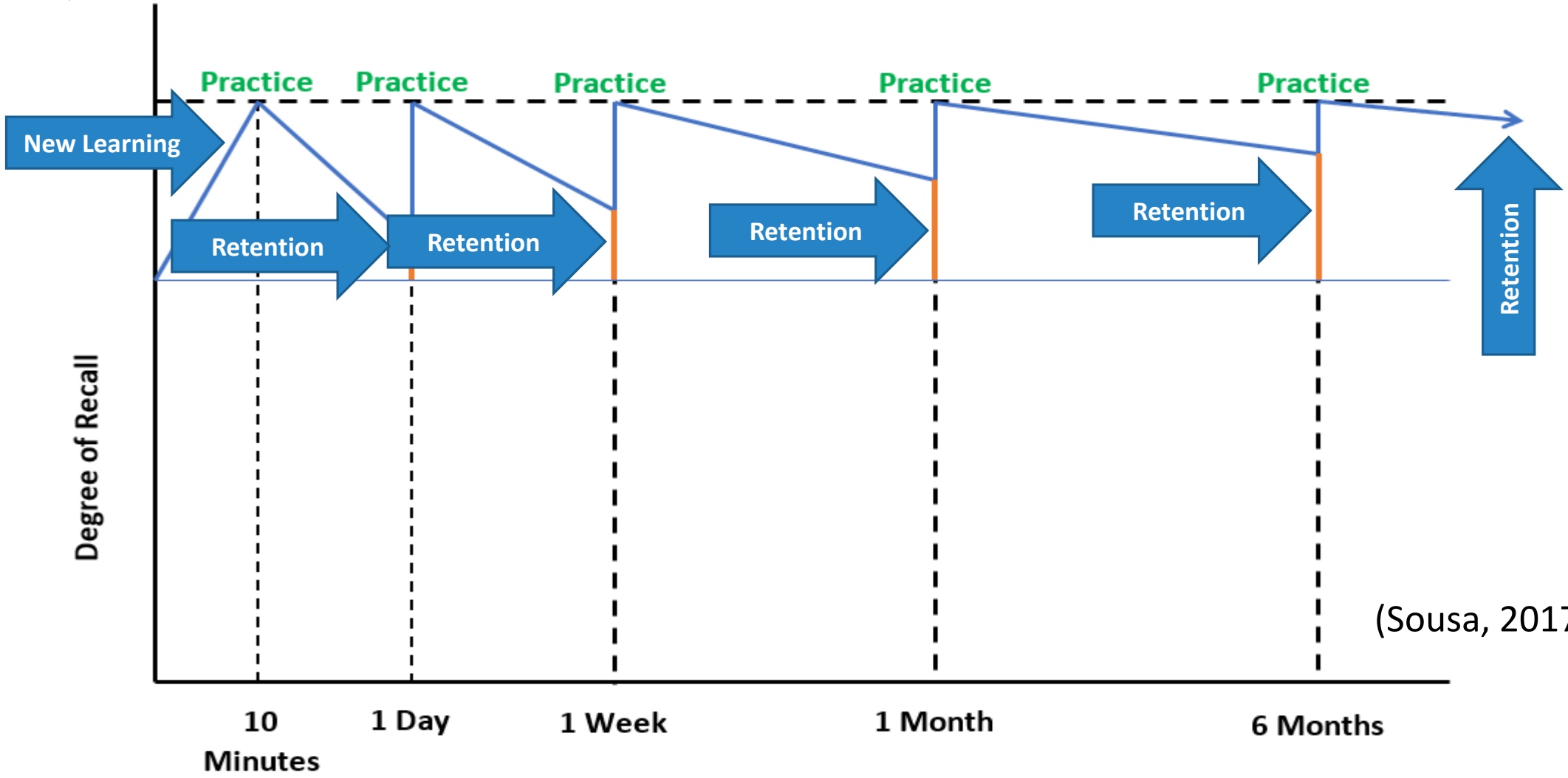
- Practice new learning during time periods that are close together
- Promotes fast learning (intermediate memory)
- The information can fade in seconds if it is not rehearsed quickly

Distributed

- Distribute reengagement/practice over longer periods of time
- Sustains meaning and consolidates learning into long-term storage
- Spiral curriculum reviews critical skills at regular intervals spanning within and over several course-levels

Sousa, D. A. (2017). *How the brain learns*. Thousand Oaks, CA: Corwin, a Sage Publishing Company.

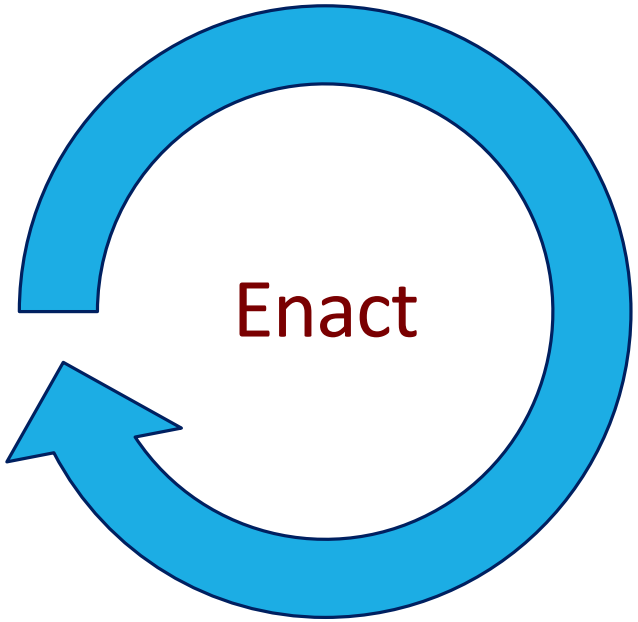
“Distributed Practice is Key for Retention”



(Sousa, 2017, p. 113)

Effective Engagement

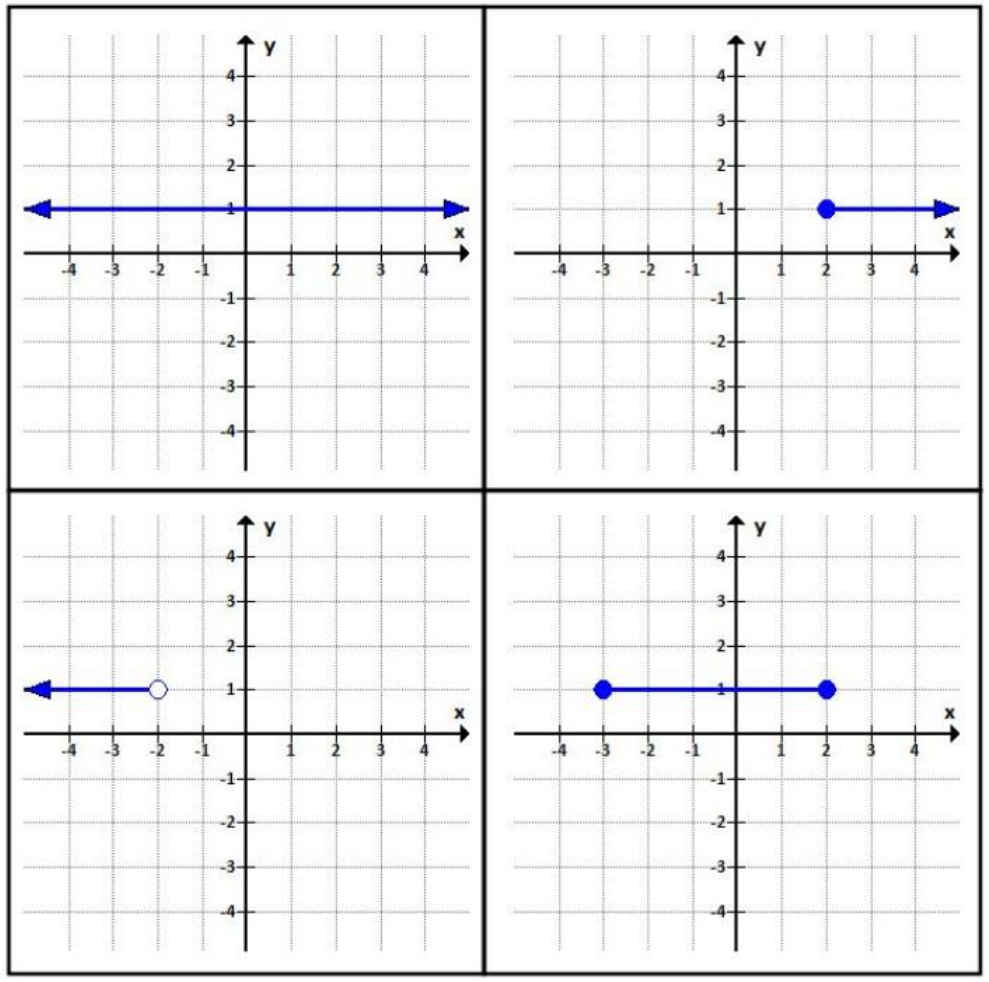
Starts with massed practice and proceeds to distributed practice for retention where students are continually practicing previously learned skills throughout the year(s) (Sousa, 2017).



Enact strategies and design learning experiences to support learning goals

- MDTP Written Response Items
- Which One Doesn't Belong
- Error Analysis
- Notice and Wonder

Which One Doesn't Belong?



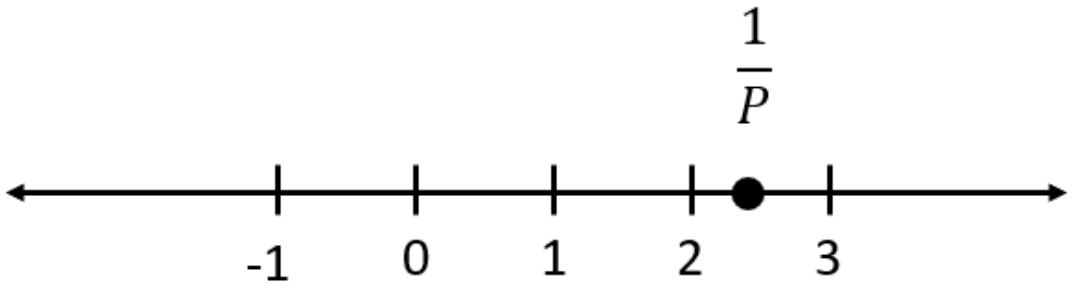
1) Design a WODB that unpacks a misconception from your topic
2) Think about bridging the content to your current course level
3) Think about having students create their own WODB

Taken from: http://wodb.ca

MDTP and Error Analysis

- MDTP's drill-down results motivate teacher conversations around error-detection.
- Error detection can be "very powerful, provided students have some modicum of knowledge and understanding about the task on which to strategize and regulate" (Hattie and Timperley, 2007, p. 86).

Error Analysis – Create a Parallel Problem



What do you notice about the point on the graph? What do you wonder?

James said the value of P is between 2 and 3.

Maria said that she knows that P cannot be 2 because ½ is not greater than 2. And she knows that P cannot be 3 because 1/3 is also not greater than 3.

However, Maria is not sure about numbers between 2 and 3 like 2.5. How can you help Maria to check values between 2 and 3 to check James' claim?

What value(s) could P be?

Resources

- MDTP Written Response Items (WRI): http://mdtp.ucsd.edu
- I Notice I Wonder: https://www.nctm.org/Classroom-Resources/Problems-of-the-Week/Extras/I-Notice-I-Wonder/
- Which One Doesn't Belong: http://wodb.ca/
- Number Talks (various sites): https://elemath.hallco.org/web/wp-content/uploads/2014/05/Number-Talks-Quick-Start-Guide.pdf
- MDTP Modules: https://mdtpmodules.org/

References

Hattie, J., & Timperley, H. (2007). The power of feedback. Review of Educational Research, 77, 81–112.

National Council of Teachers of Mathematics. 2014. *Principles to actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics.

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