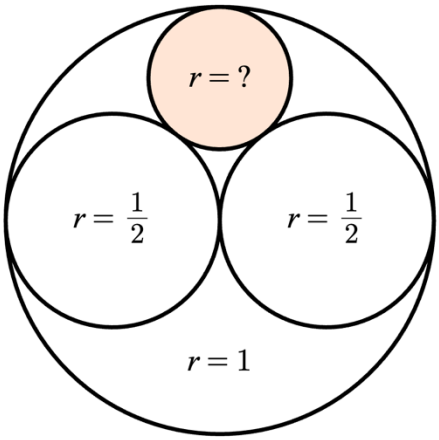
In exploring arrangements of tangent circles, the statistician Allan R. Wilks investigated a simple geometric puzzle involving circles packed inside a larger circle:

"The route to the discovery began with a high school geometry problem. In 1998, during a lull in a conference in Germany, Wilks started talking with a colleague who was pondering his daughter's homework assignment. The question concerned a pattern made up of two identical circles that fit side by side inside a third circle. The assignment was to find the radius of a fourth circle nestled between the outer circle and the two inner circles.

After Wilks had returned to the United States, he found the three-circle pattern still on his mind. In drawing it, he could easily insert smaller circles into the empty spaces among the larger circles. Each new circle would be as large as possible without overlapping the circles already present."

From: https://www.sciencenews.org/article/circle-game



Find the radius of a fourth circle nestled between the outer circle of radius 1, and the two inner circles, each of radius 1/2.

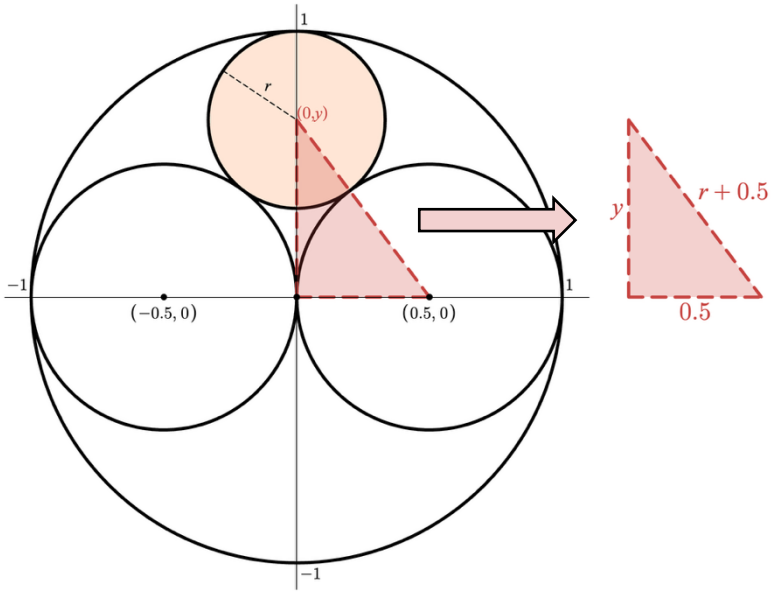
Solution:

Let's assume the larger circle with radius 1 is centered at the origin. The centers of the two inner circles, each with radius $1/2$, can be placed at $(-0.5, 0)$ and $(0.5, 0)$ so that they are tangent to each other at the origin and tangent to the outer circle at $(-1, 0)$ and $(1, 0)$.

Let the fourth circle have an unknown radius r and center $(0, y)$ (by symmetry, its x-coordinate must be 0). This circle must satisfy two conditions:

1. It must be tangent to the outer circle at $(0, 1)$, so

$$y = 1 - r$$



2. It must be tangent to an inner circle, so the distance from its center $(0, y)$ to the center of one inner circle, say $(0.5, 0)$, must be the sum of their radii, $r + 0.5$, and

$$y^2 + 0.5^2 = (r + 0.5)^2$$

by the Pythagorean Theorem.

We can solve this system of equations to find the value of r. Substitute $y = 1 - r$ into $y^2 + 0.5^2 = (r + 0.5)^2$:

$$(1 - r)^2 + 0.5^2 = (r + 0.5)^2$$

$$1 - 2r + r^2 + 0.25 = r^2 + r + 0.25$$

$$1 = 3r$$

$$r = \frac{1}{3}$$

Thus, the radius of the fourth circle is $1/3$.